

Trade with Nominal Rigidities: Understanding the Unemployment and Welfare Effects of the China Shock

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Appendix for Online Publication

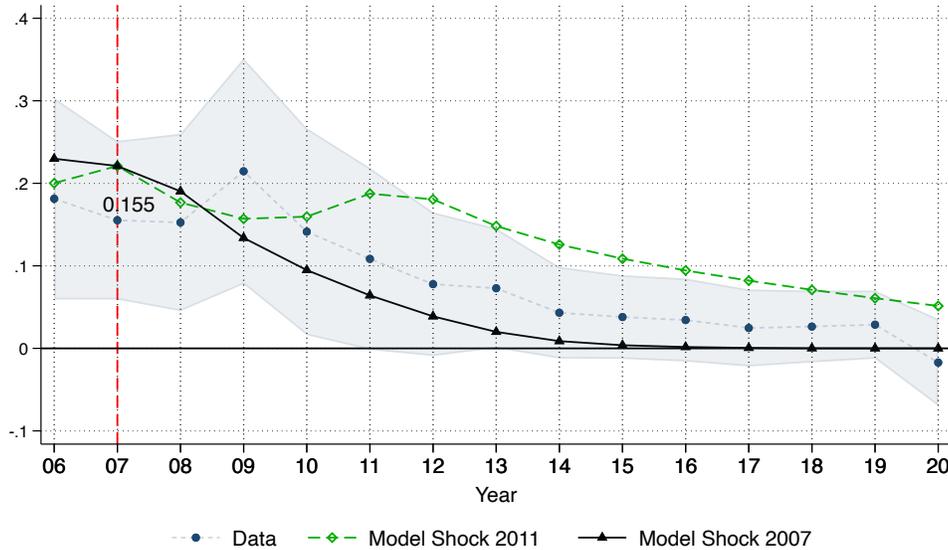
March 2025

This online appendix supplements the paper “Trade with Nominal Rigidities: Understanding the Unemployment and Welfare Effects of the China Shock” with the following material:

- Appendix [A](#) provides additional tables and figures referenced in the main text.
- Appendix [B](#) presents the details of the model presented in Section 3.
- Appendix [C](#) contains more details about the construction of the datasets necessary for the model’s calibration, which are described in Section 4.
- Appendix [D](#) presents theory and quantification for the alternative exposure measure discussed in Section 8.

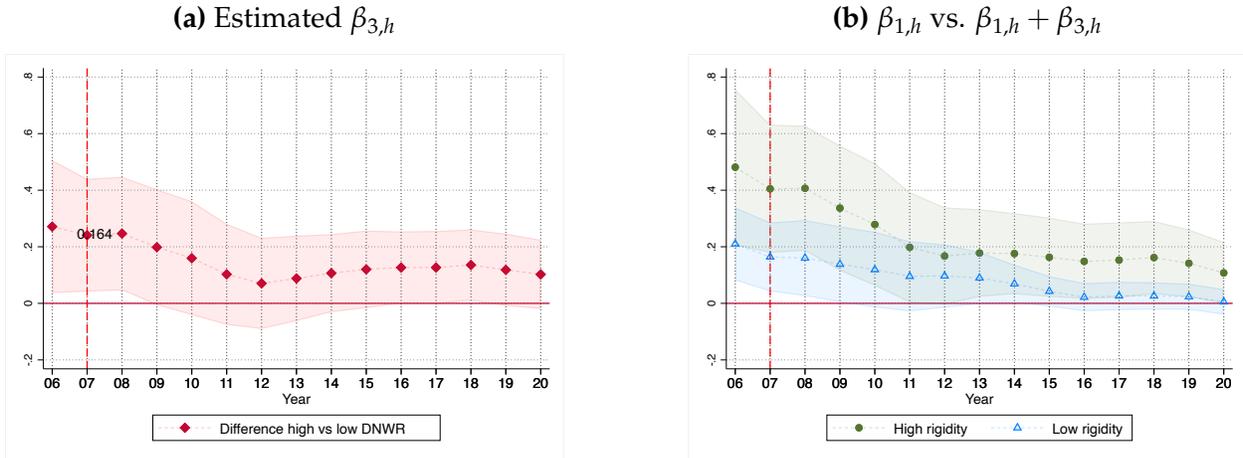
A Additional Figures and Tables

Figure A.1: Baseline empirical analysis using an alternative construction of the unemployment-to-population ratio



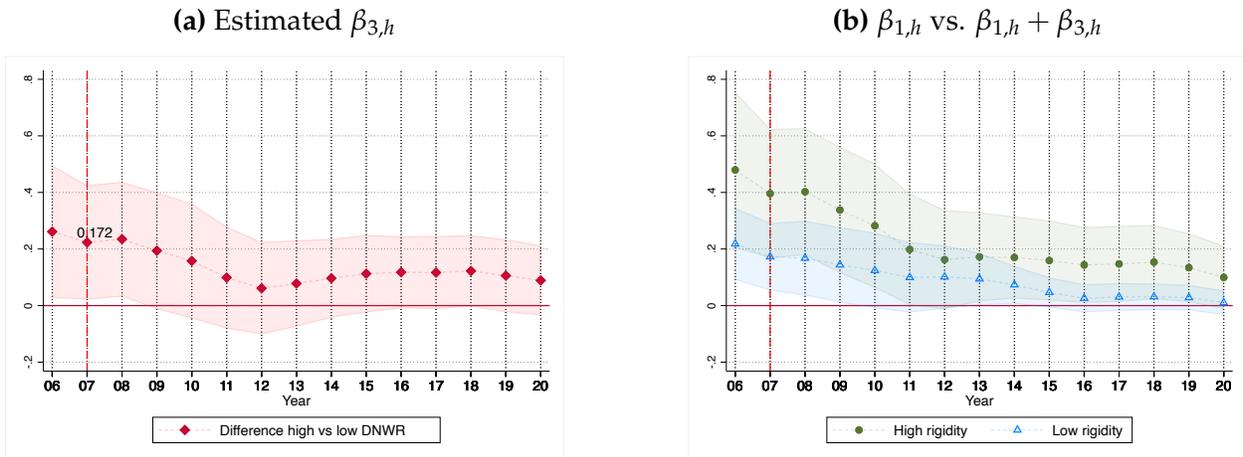
Notes: The figure reports three lines. The blue line with circular markers shows the two-stage least squares coefficient estimates for β_{1h} in equation (1) where the outcome variable is the unemployment-to-population ratio, and this variable is constructed using BLS data for unemployment and SEER data for population (in contrast to the ACS data used in ADH). The shaded area represents the 95% confidence interval. The regression estimates are analogous to those in Figure 1, differing only in the data source used to compute the unemployment-to-population ratio. The green dashed line displays the effects in our model when the China shock lasts between 2001 and 2011 (labeled as “Model Shock 2011”), while the black line with triangular markers displays the effects in our baseline model when the China shock lasts between 2001 and 2007 (labeled as “Model Shock 2007”).

Figure A.2: China Shock on unemployment in CZs with high vs low DNWR. Share of individuals with negative wage changes in state s is below the median across all states



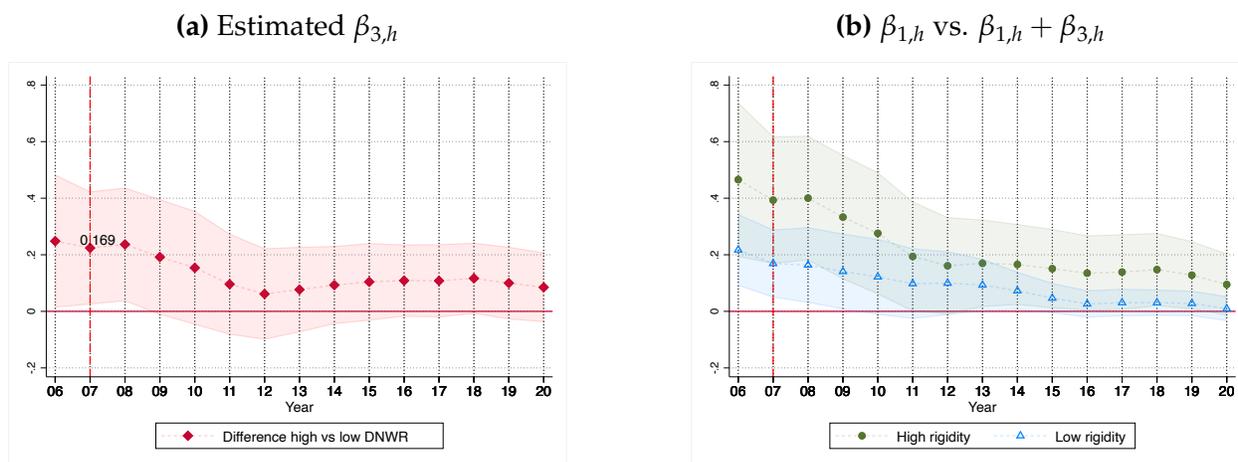
Note: The panels report two-stage least squares coefficient estimates in equation (2) and 95 percent confidence intervals for these estimates. Coefficients in each year come from a separate IV regression following equation (2).

Figure A.3: China Shock on unemployment in CZs with high vs low DNWR. Share of individuals with negative wage changes among non-zero wage changes in state s is below the mean across all states



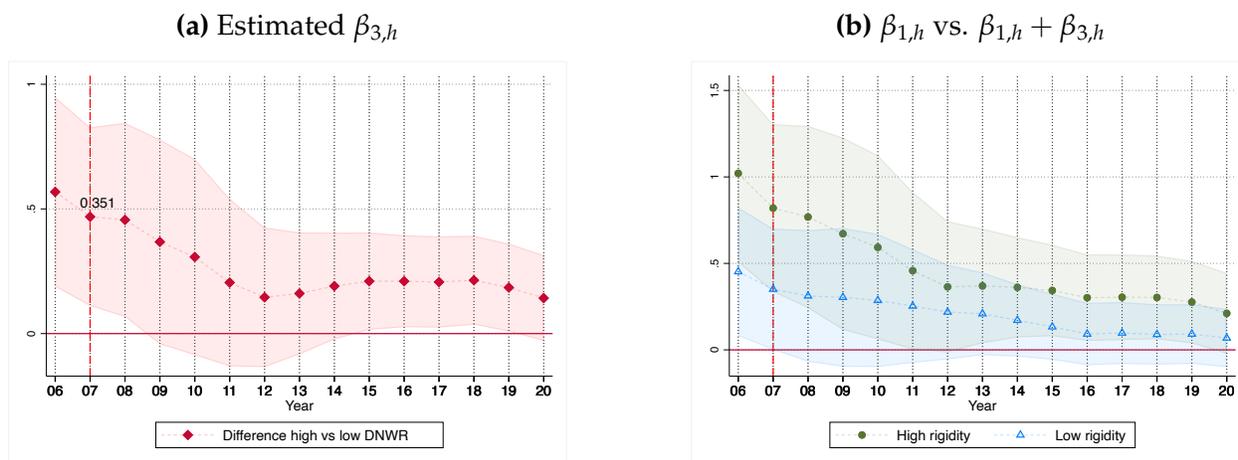
Note: The panels report two-stage least squares coefficient estimates in equation (2) and 95 percent confidence intervals for these estimates. Coefficients in each year come from a separate IV regression following equation (2).

Figure A.4: China Shock on unemployment in CZs with high vs low DNWR. Share of individuals with negative wage changes among non-zero wage changes in state s is below the median across all states



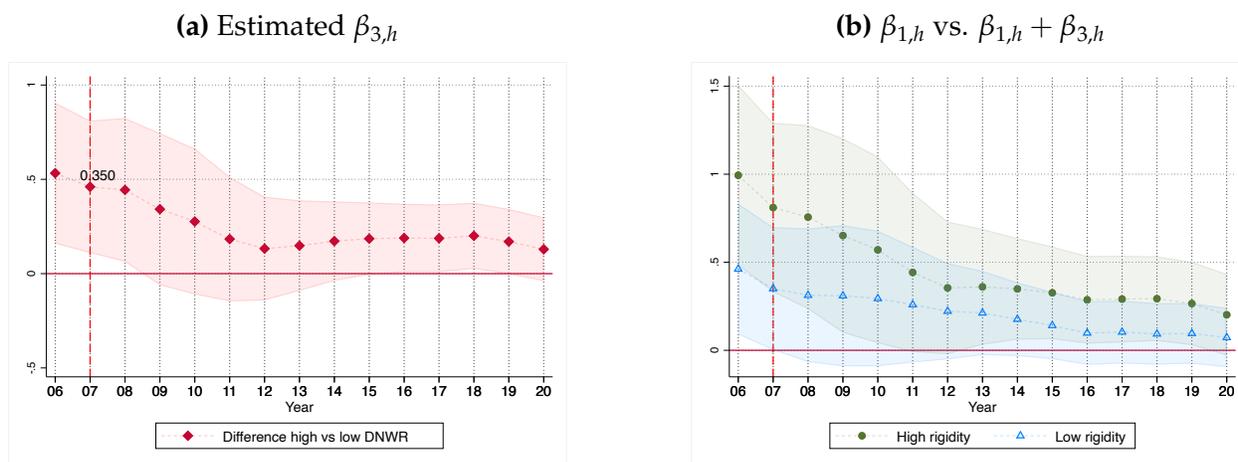
Note: The panels report two-stage least squares coefficient estimates in equation (2) and 95 percent confidence intervals for these estimates. Coefficients in each year come from a separate IV regression following equation (2).

Figure A.5: China Shock on unemployment in States with high vs low DNWR. Share of individuals with negative wage changes in state s is below the mean across all states



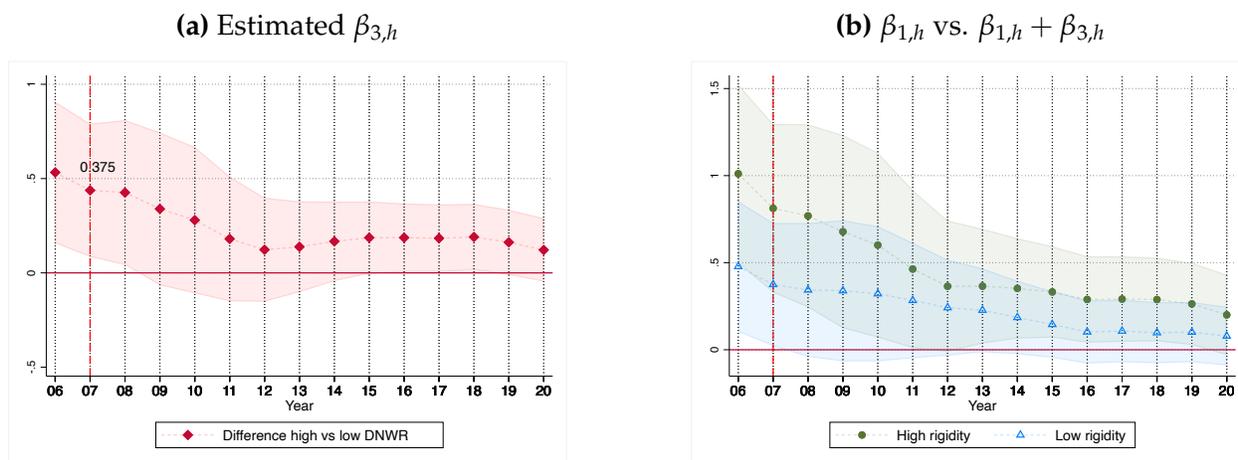
Note: The panels report two-stage least squares coefficient estimates in equation (2) and 95 percent confidence intervals for these estimates. Coefficients in each year come from a separate IV regression following equation (2).

Figure A.6: China Shock on unemployment in States with high vs low DNWR. Share of individuals with negative wage changes in state s is below the median across all states



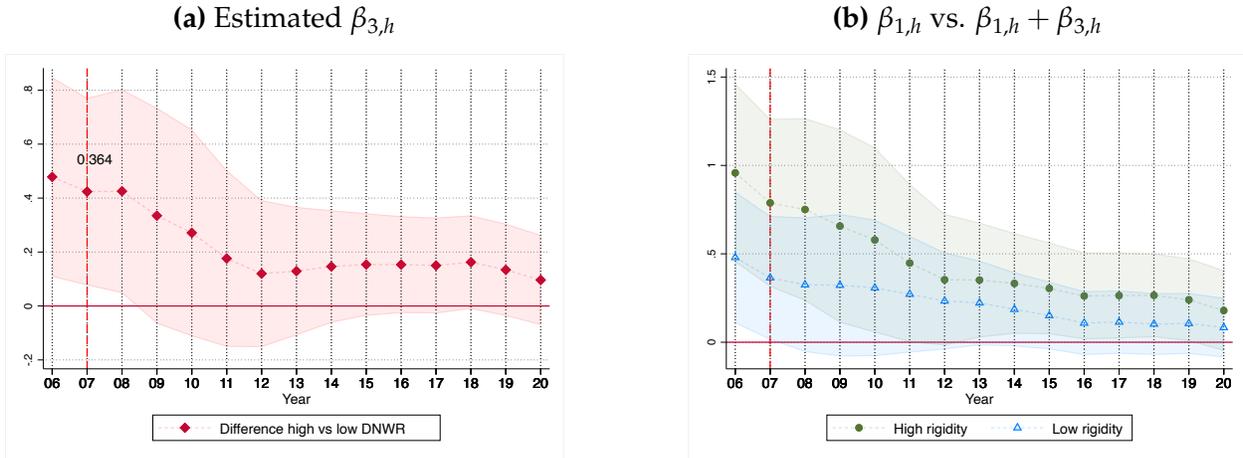
Note: The panels report two-stage least squares coefficient estimates in equation (2) and 95 percent confidence intervals for these estimates. Coefficients in each year come from a separate IV regression following equation (2).

Figure A.7: China Shock on unemployment in States with high vs low DNWR. Share of individuals with negative wage changes among non-zero wage changes in state s is below the mean across all states



Note: The panels report two-stage least squares coefficient estimates in equation (2) and 95 percent confidence intervals for these estimates. Coefficients in each year come from a separate IV regression following equation (2).

Figure A.8: China Shock on unemployment in States with high vs low DNWR. Share of individuals with negative wage changes among non-zero wage changes in state s is below the median across all states



Note: The panels report two-stage least squares coefficient estimates in equation (2) and 95 percent confidence intervals for these estimates. Coefficients in each year come from a separate IV regression following equation (2).

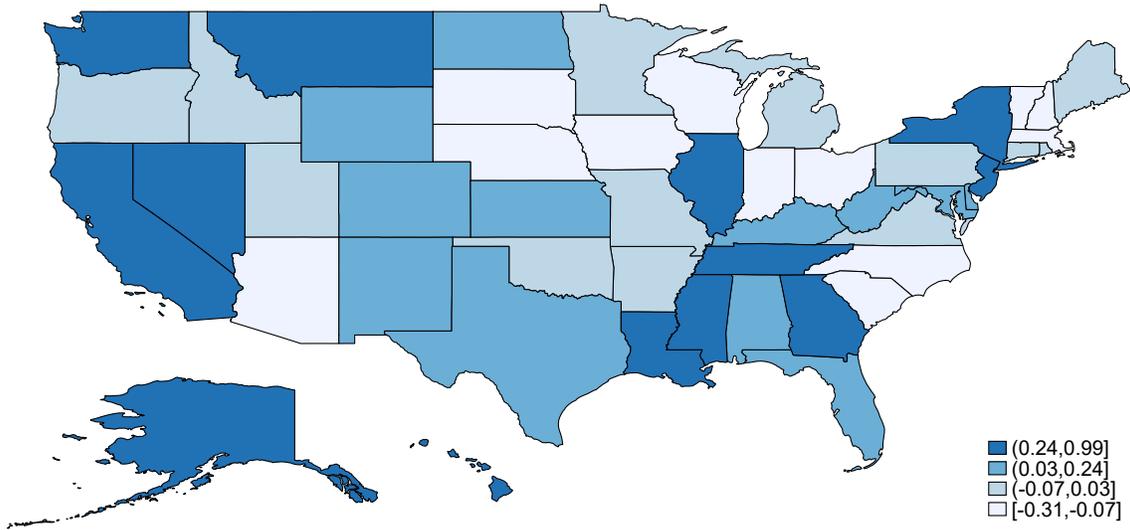
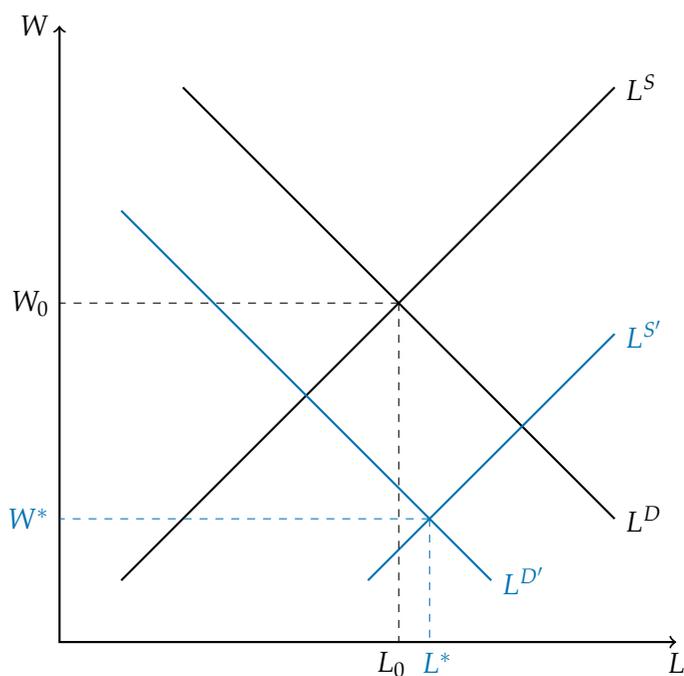


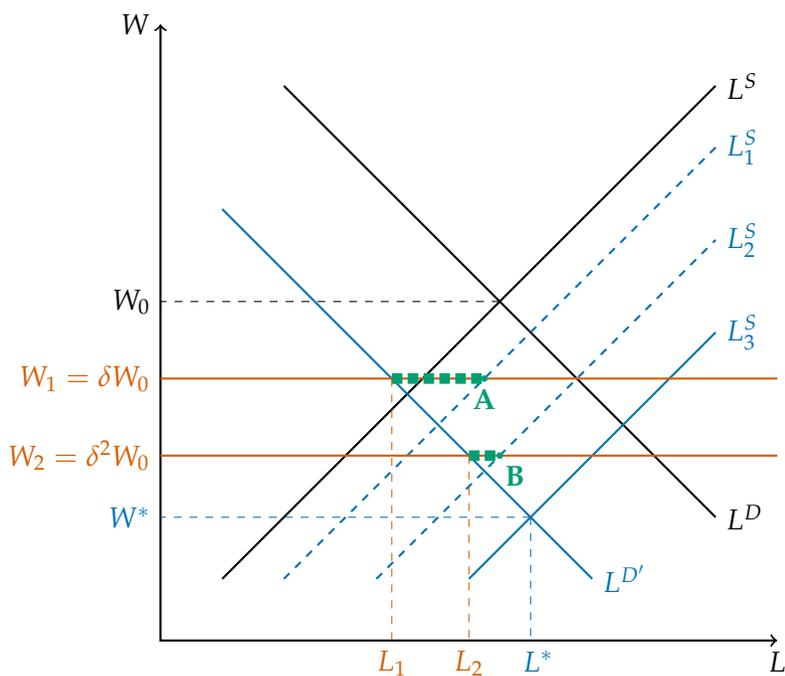
Figure A.9: Welfare change from the China shock across U.S. states in the baseline specification, in percent.

A.1 Additional Intuition for the Results

We illustrate the mechanisms discussed in Section 6.2 in Figure A.10. Both panels have the nominal wage in the vertical axis and employment in the horizontal axis. The China shock leads to a fall in producer prices, shifting labor demand down. At the same time, the shock also leads to



(a) Case without DNWR



(b) Case with DNWR

Figure A.10: Illustration of wage and employment effects, with and without DNWR. The nominal wage is in the vertical axis, hence price movements results in shifts in the labor supply curve. Employment is in the horizontal axis.

a decline in consumer prices, shifting the labor supply to the right. Panel (a) presents the results without nominal frictions. The final result is a fall in the nominal wage from W_0 to W^* , a fall in prices from P_0 to P^* (not illustrated), an increase in the real wage from W_0/P_0 to W^*/P^* (prices fall more than nominal wages), and an increase in the amount of labor supplied from L_0 to L^* .

Panel (b) of Figure A.10 shows the adjustment in the presence of DNWR assuming that $\delta^3 W_0 < W^* < \delta^2 W_0$. In the first year, the nominal wage only falls from W_0 to $W_1 \equiv \delta W_0$ and employment falls from L_0 to L_1 , as determined by the demand curve. Since the nominal wage does not fully adjust in the first year, the fall in consumer prices is also smaller than in the frictionless case, and hence the labor supply curve only moves from L^S to L_1^S . The gap between the labor supplied at point A and labor demanded L_1 is the level of unemployment. In the second year, nominal wages adjust further down (to $W_2 \equiv \delta W_1 = \delta^2 W_0$), the labor supply curve moves to L_2^S , employment increases from L_1 to L_2 , labor supplied moves from point A to point B, and unemployment decreases. In the third year, nominal wages finally adjust fully and there is no more unemployment.

B Model Details

B.1 Production

Technology to produce the differentiated good of industry s in region i at time t is

$$Y_{i,s,t} = \left(\phi_{i,s}^{-\phi_{i,s}} \prod_{k=1}^S \phi_{i,ks}^{-\phi_{i,ks}} \right) A_{i,s,t} L_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S M_{i,ks,t}^{\phi_{i,ks}}$$

where $M_{i,ks,t}$ is the quantity of the composite good of industry k used in region i to produce in sector s at time t , $\phi_{i,s}$ is the labor share in region i , sector s , $\phi_{i,ks}$ is the share of inputs that sector s uses from sector k in region i , and $1 - \phi_{i,s} = \sum_{k=1}^S \phi_{i,ks}$. The resource constraint for the composite good produced in region j , sector k , at time t is

$$M_{j,k,t} = C_{j,k,t} + \sum_{s=1}^S M_{j,ks,t}.$$

In turn, the resource constraint for good s produced by region i is $Y_{i,s,t} = \sum_{j=1}^I \tau_{ij,s,t} Y_{ij,s,t}$. The composite in sector k is produced according to

$$M_{j,k,t} = \left(\sum_{i=1}^I Y_{ij,k,t}^{\frac{\sigma_k-1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k-1}}.$$

Let $P_{i,s,t}$ be the price of $M_{i,s,t}$, $p_{ij,s,t}$ be the price of $Y_{i,s,t}$ in j at time t , and $W_{i,s,t}$ be the nominal wage in region i , sector s , at time t . We know that $p_{ii,s,t} = A_{i,s,t}^{-1} W_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S P_{i,k,t}^{\phi_{i,ks}}$, $p_{ij,s,t} = \tau_{ij,s,t} p_{ii,s,t}$, and $P_{j,s,t} = \left(\sum_{i=1}^I p_{ij,s,t}^{1-\sigma_s} \right)^{1/(1-\sigma_s)}$. Combining these we obtain:

$$P_{j,s,t}^{1-\sigma_s} = \sum_{i=1}^I \left(\tau_{ij,s,t} A_{i,s,t}^{-1} W_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S P_{i,k,t}^{\phi_{i,ks}} \right)^{1-\sigma_s},$$

The price of final output in region j at time t is given by $P_{j,t} = \prod_{s=1}^S P_{j,s,t}^{\alpha_{j,s}}$. Multiplying the resource constraint for $M_{j,k,t}$ by $P_{j,k,t}$ we get

$$Z_{j,k,t} = P_{j,k,t} C_{j,k,t} + \sum_{s=1}^S P_{j,k,t} M_{j,ks,t},$$

where $Z_{j,k,t} \equiv P_{j,k,t} M_{j,k,t}$ denotes the total expenditure of region j in industry k . Let the share of that expenditure spent on imports from i be $\lambda_{ij,k,t} \equiv \frac{p_{ij,k,t} Y_{ij,k,t}}{Z_{j,k,t}}$. We know that

$$\lambda_{ij,k,t} = \frac{p_{ij,k,t}^{1-\sigma_k}}{\sum_l p_{lj,k,t}^{1-\sigma_k}} = \frac{p_{ij,k,t}^{1-\sigma_k}}{P_{j,k,t}^{1-\sigma_k}} = \frac{\left(\tau_{ij,k,t} A_{i,k,t}^{-1} W_{i,k,t}^{\phi_{i,k}} \prod_{s=1}^S P_{i,s,t}^{\phi_{i,sk}} \right)^{1-\sigma_k}}{\sum_{r=1}^I \left(\tau_{rj,k,t} A_{r,k,t}^{-1} W_{r,k,t}^{\phi_{r,k}} \prod_{s=1}^S P_{r,s,t}^{\phi_{r,sk}} \right)^{1-\sigma_k}}.$$

Let $R_{i,k,t} = p_{ii,k,t} Y_{i,k,t}$ represent the sales of good k by region i . Multiplying the resource constraint for $Y_{i,k,t}$ above by $p_{ii,k,t}$ we get $p_{ii,k,t} Y_{i,k,t} = \sum_{j=1}^I \tau_{ij,k,t} p_{ii,k,t} Y_{ij,k,t}$, and hence $R_{i,k,t} = \sum_{j=1}^I \lambda_{ij,k,t} Z_{j,k,t}$. Plugging in from the resource constraint for $Z_{j,k,t}$ we have

$$R_{i,k,t} = \sum_{j=1}^I \lambda_{ij,k,t} \left(P_{j,k,t} C_{j,k,t} + \sum_s P_{j,k,t} M_{j,ks,t} \right).$$

Note that $P_{j,k,t} M_{j,ks,t} = \phi_{j,ks} R_{j,s,t}$. Additionally, the total amount available for consumption in region j at time t is the sum of total labor income (denoted $I_{j,t}$, notice $I_{j,t} \equiv \sum_{k=1}^S W_{j,k,t} L_{j,k,t}$) and the deficit (denoted $D_{j,t}$). So we get $P_{j,k,t} C_{j,k,t} = \alpha_{j,k} (I_{j,t} + D_{j,t})$, hence

$$R_{i,k,t} = \sum_{j=1}^I \lambda_{ij,k,t} \left(\alpha_{j,k} (I_{j,t} + D_{j,t}) + \sum_s \phi_{j,ks} R_{j,s,t} \right).$$

We know that a fraction $\phi_{i,k}$ of $R_{i,k,t}$ is paid to labor, hence $W_{i,k,t}L_{i,k,t} = \phi_{i,k}R_{i,k,t}$.

B.2 Labor Supply

As mentioned in the text, an agent's utility in region j , sector s , at time t is given by

$$v_{j,s,t} = U_{j,s,t} + \max_{\{i,k\}_{i=1,k=0}^{I,S}} \{\beta \mathbb{E}(v_{i,k,t+1}) - \varphi_{ji,sk} + \epsilon_{i,k,t}\},$$

with the joint distribution of vector ϵ being i.i.d over time and nested Gumbel,

$$F(\epsilon) = \exp \left(- \sum_{i=1}^I \left(\sum_{k=0}^S \exp(-\epsilon_{i,k}/\nu) \right)^{\nu/\kappa} \right)$$

with $\nu \leq \kappa$. If there is an strict inequality such that $\nu < \kappa$, that means that the elasticity across sectors ($1/\nu$) is greater than the elasticity across locations ($1/\kappa$). Denote $V_{i,k,t+1} \equiv \mathbb{E}[v_{i,k,t+1}]$. In this appendix, we will prove two main results. First, the probability that an agent in js will choose to move to ik conditional on moving to region i is

$$\mu_{ji,sk|i,t} = \frac{\exp(\beta V_{i,k,t+1} - \varphi_{ji,sk})^{1/\nu}}{\sum_{h=0}^S \exp(\beta V_{i,h,t+1} - \varphi_{ji,sh})^{1/\nu}},$$

while the probability that an agent in js will move to any sector in region i is

$$\mu_{ji,s\#,t} = \frac{\left(\sum_{h=0}^S \exp(\beta V_{i,h,t+1} - \varphi_{ji,sh})^{1/\nu} \right)^{\nu/\kappa}}{\sum_{m=1}^I \left(\sum_{h=0}^S \exp(\beta V_{m,h,t+1} - \varphi_{jm,sh})^{1/\nu} \right)^{\nu/\kappa}}.$$

Second,

$$\mathbb{E} \left[\max_{\{i,k\}_{i=1,k=0}^{I,S}} \{\beta \mathbb{E}(v_{i,k,t+1}) - \varphi_{ji,sk} + \epsilon_{i,k,t}\} \right] = \ln \left(\sum_{i=1}^I \left(\sum_{k=0}^S \exp(\beta V_{i,k,t+1} - \varphi_{ji,sk})^{1/\nu} \right)^{\nu/\kappa} \right)^{\kappa} + \gamma\kappa,$$

where γ is the Euler-Mascheroni constant. The previous expression implies

$$V_{j,s,t} = U_{j,s,t} + \ln \left(\sum_{i=1}^I \left(\sum_{k=0}^S \exp(\beta V_{i,k,t+1} - \varphi_{ji,sk})^{1/\nu} \right)^{\nu/\kappa} \right)^{\kappa} + \gamma\kappa.$$

To show the first result, note that an agent that is in market js at time t will choose to switch

to ik if and only if the following expression holds for all mh :

$$\beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t} \geq \beta V_{m,h,t+1} - \varphi_{jm,sh} + \epsilon_{m,h,t},$$

which is equivalent to $\epsilon_{m,h,t} \leq \nu x_{im,kh} + \epsilon_{i,k,t}$, where

$$x_{im,kh} \equiv \frac{\beta (V_{i,k,t+1} - V_{m,h,t+1}) - (\varphi_{ji,sk} - \varphi_{jm,sh})}{\nu}.$$

Denoting

$$\Phi_{j,s,t} \equiv \mathbb{E} \left[\max_{\{i,k\}_{i=1,k=0}^{I,S}} \{ \beta \mathbb{E}(\nu_{i,k,t+1}) - \varphi_{ji,sk} + \epsilon_{i,k,t} \} \right]$$

We know that

$$\Phi_{j,s,t} = \sum_{i=1}^I \sum_{k=0}^S \int_{-\infty}^{+\infty} (\beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t}) G_{ik}(\epsilon_{i,k,t}, x_{i,k,t}) d\epsilon_{i,k,t}$$

where $G_{ik}(\epsilon_{i,k,t}, x_{i,k,t})$ is the partial derivative of $F(\cdot)$ w.r.t. to the ik element of the vector ϵ , with the ik element of the vector evaluated at $\epsilon_{i,k,t}$ and the generic element in position mh of the vector evaluated at $\nu x_{im,kh,t} + \epsilon_{i,k,t}$. Given our function $F(\epsilon)$ above, the partial derivative w.r.t the element in position ik is

$$\frac{\partial F(\epsilon)}{\partial \epsilon_{i,k}} = \frac{1}{\kappa} \left(\sum_h \exp(-\epsilon_{i,h}/\nu) \right)^{\nu/\kappa-1} \exp(-\epsilon_{i,k}/\nu) \exp \left[- \sum_m \left(\sum_h \exp(-\epsilon_{m,h}/\nu) \right)^{\nu/\kappa} \right]$$

We then have

$$\begin{aligned} G_{ik}(\epsilon_{i,k,t}, x_{i,k,t}) &= \frac{1}{\kappa} \left(\sum_h \exp(-x_{ii,kh,t}) \right)^{\nu/\kappa-1} \exp(-\epsilon_{i,k,t}/\kappa) \\ &\cdot \exp \left[- \exp(-\epsilon_{i,k,t}/\kappa) \sum_m \left(\sum_h \exp(-x_{im,kh,t}) \right)^{\nu/\kappa} \right] \end{aligned}$$

where we have used the fact that $x_{ii,kk,t} = 0$. Integrating this over $\epsilon_{i,k,t}$ yields

$$\begin{aligned} \int_{-\infty}^{+\infty} G_{ik}(\epsilon_{i,k,t}, x_{i,k,t}) d\epsilon_{i,k,t} &= \frac{(\sum_h \exp(-x_{ii,kh,t}))^{\nu/\kappa-1}}{\sum_m (\sum_h \exp(-x_{im,kh,t}))^{\nu/\kappa}} \int_{-\infty}^{+\infty} \frac{1}{\kappa} \exp(-\epsilon_{i,k,t}/\kappa) \\ &\cdot T \exp \left[- \exp(-\epsilon_{i,k,t}/\kappa) T \right] d\epsilon_{i,k,t}, \end{aligned}$$

where

$$T \equiv \sum_m \left(\sum_h \exp(-x_{im, kh, t}) \right)^{v/\kappa}.$$

But note that

$$\int_{-\infty}^{+\infty} \frac{1}{\kappa} \exp(-\epsilon_{i, k, t}/\kappa) T \exp \left[-\exp(-\epsilon_{i, k, t}/\kappa) T \right] d\epsilon_{i, k, t} = 1,$$

because the integrand is the density associated with $\exp(-\exp(-\epsilon_{i, k, t}/\kappa) T)$, a univariate Gumbel. Hence, the previous expression simplifies to

$$\int_{-\infty}^{\infty} G_{ik}(\epsilon_{i, k, t}, x_{i, k, t}) d\epsilon_{i, k, t} = \frac{\exp(\beta V_{i, k, t+1} - \varphi_{ji, sk})^{1/v} \left(\sum_h \exp(\beta V_{i, h, t+1} - \varphi_{ji, sh}) \right)^{1/v} v/\kappa}{\sum_h \exp(\beta V_{i, h, t+1} - \varphi_{ji, sh})^{1/v} \sum_m \left(\sum_h \exp(\beta V_{m, h, t+1} - \varphi_{jm, sh}) \right)^{1/v} v/\kappa}$$

It is easy to see that the first fraction is $\mu_{ji, sk|i, t}$, while the second one is $\mu_{ji, s\#t}$.

Now we want to solve for

$$\mathbb{E} \left[\max_{\{i, k\}_{i=1, k=0}^{t, S}} \{\beta V_{i, k, t+1} - \varphi_{ji, sk} + \epsilon_{i, k, t}\} \right]$$

Let's compute

$$\mathbb{E} \left[\beta V_{i, k, t+1} - \varphi_{ji, sk} + \epsilon_{i, k, t} \mid \arg \max_{mh} \{\beta V_{m, h, t+1} - \varphi_{jm, sh} + \epsilon_{m, h, t}\} = ik \right]$$

To do this, first note that the joint probability that $\beta V_{i, k, t+1} - \varphi_{ji, sk} + \epsilon_{i, k, t} \leq a$ while at the same time $\arg \max_{mh} \{\beta V_{m, h, t+1} - \varphi_{jm, sh} + \epsilon_{m, h, t}\} = ik$, is

$$\begin{aligned} & \int_{-\infty}^{a - (\beta V_{i, k, t+1} - \varphi_{ji, sk})} G_{ik}(\epsilon_{i, k, t}, x_{i, k, t}) d\epsilon_{i, k, t} \\ &= \frac{(\sum_h \exp(-x_{ii, kh}))^{v/\kappa - 1}}{\sum_m (\sum_h \exp(-x_{im, kh}))^{v/\kappa}} \int_{-\infty}^{a - (\beta V_{i, k, t+1} - \varphi_{ji, sk})} \frac{1}{\kappa} T \exp(-z/\kappa) \exp(-T \exp(-z/\kappa)) dz \end{aligned}$$

A change of variables with $y = \exp(z)$ implies that $dy/y = dz$ and

$$\begin{aligned} & \int_{-\infty}^{a - (\beta V_{i, k, t+1} - \varphi_{ji, sk})} \frac{1}{\kappa} T \exp(-z/\kappa) \exp(-T \exp(-z/\kappa)) dz \\ &= \exp(-T \exp[(\beta V_{i, k, t+1} - \varphi_{ji, sk})/\kappa]) \exp(-a/\kappa) \end{aligned}$$

Thus, the **joint** probability we are interested in is

$$\frac{(\sum_h \exp(-x_{ii,kh}))^{v/\kappa-1}}{\sum_m (\sum_h \exp(-x_{im,kh}))^{v/\kappa}} \exp(-T \exp[(\beta V_{i,k,t+1} - \varphi_{ji,sk}) / \kappa]) \exp(-a/\kappa)$$

and hence the probability of $(\beta V_{i,k,t+1} - \varphi^{ji,sk} + \epsilon_{i,k,t} \leq a)$ **conditional on**

$$\arg \max_{mh} \{\beta V_{m,h,t+1} - \varphi_{jm,sh} + \epsilon_{m,h,t}\} = ik,$$

is

$$\exp(-\tilde{T} \exp(-a/\kappa)),$$

where now

$$\tilde{T} \equiv T \exp[(\beta V_{i,k,t+1} - \varphi_{ji,sk}) / \kappa].$$

In turn, this implies that

$$\begin{aligned} & \mathbb{E} \left[\beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t} \mid \arg \max_{mh} \{\beta V_{m,h,t+1} - \varphi_{jm,sh} + \epsilon_{m,h,t}\} = ik \right] \\ &= \int_{-\infty}^{+\infty} ad \exp \left(- \exp \left(- \frac{(a - \ln \tilde{T}^\kappa)}{\kappa} \right) \right), \end{aligned}$$

where we have used

$$\tilde{T} \exp(-a/\kappa) = \exp \left(- \frac{(a - \ln \tilde{T}^\kappa)}{\kappa} \right).$$

This is the expectation of a variable distributed Gumbel with location parameter $\mu = \ln \tilde{T}^\kappa$ and scale parameter $\beta = \kappa$. But we know that the expectation of a variable distributed Gumbel with μ and β is $\mu + \beta\gamma$, where γ is the Euler-Mascheroni constant, hence we have

$$\int_{-\infty}^{\infty} ad \exp \left(-\tilde{T} (\exp a)^{-1/\kappa} \right) = \ln \tilde{T}^\kappa + \gamma\kappa.$$

This implies that

$$\begin{aligned} & \mathbb{E} \left[\beta V_{i,k,t+1} - \varphi_{ji,sk} + \epsilon_{i,k,t} \mid \arg \max_{mh} \{\beta V_{m,h,t+1} - \varphi_{jm,sh} + \epsilon_{m,h,t}\} = ik \right] \\ &= \ln \left(\sum_m \left(\sum_h \exp(\beta V_{m,h,t+1} - \varphi_{jm,sh})^{1/v} \right)^{v/\kappa} \right) + \gamma\kappa \end{aligned}$$

Since this does not depend on ik , then we have

$$\mathbb{E} \left[\max_{\{i,k\}_{i=1}^I, \{s\}_{k=0}^S} \{ \beta \mathbb{E}(v_{i,k,t+1}) - \varphi_{ji,sk} + \epsilon_{i,k,t} \} \right] = \ln \left(\sum_{i=1}^I \left(\sum_{k=0}^S \exp(\beta V_{i,k,t+1} - \varphi_{ji,sk})^{1/v} \right)^{\nu/\kappa} \right)^{\kappa} + \gamma\kappa,$$

as we wanted to show.

B.3 Equilibrium in Relative Time Changes (Dots)

Now we will describe the equilibrium equations in relative changes from one period to the next. We use the notation $\dot{x}_t = x_t/x_{t-1}$. We start by deriving the dot equations for the labor market block of the economy. We will denote $u_{j,s,t} \equiv \exp(V_{j,s,t})$ and assume that the utility function takes log form. We have,

$$\begin{aligned} \frac{\mu_{ji,sk|i,t+1}}{\mu_{ji,sk|i,t}} &= \frac{\exp(\beta V_{i,k,t+2} - \varphi_{ji,sk})^{1/v} / \exp(\beta V_{i,k,t+1} - \varphi_{ji,sk})^{1/v}}{\sum_{h=0}^S \exp(\beta V_{i,h,t+2} - \varphi_{ji,sh})^{1/v} / \sum_{h'=0}^S \exp(\beta V_{i,h',t+1} - \varphi_{ji,sh'})^{1/v}} \\ &= \frac{\exp(V_{i,k,t+2} - V_{i,k,t+1})^{\beta/v}}{\sum_{h=0}^S \mu_{ji,sh|i,t} \exp(V_{i,h,t+2} - V_{i,h,t+1})^{\beta/v}} \end{aligned}$$

while

$$\frac{\mu_{ji,s\#,t+1}}{\mu_{ji,s\#,t}} = \frac{\left(\sum_{h=0}^S \exp(V_{i,h,t+2} - V_{i,h,t+1})^{\beta/v} \mu_{ji,sh|i,t} \right)^{\nu/\kappa}}{\sum_{m=1}^I \mu_{jm,s\#,t} \left(\sum_{h=0}^S \exp(V_{m,h,t+2} - V_{m,h,t+1})^{\beta/v} \mu_{jm,sh|m,t} \right)^{\nu/\kappa}}.$$

Since $u_{j,s,t} \equiv \exp(V_{j,s,t})$ then

$$\begin{aligned} \dot{u}_{j,s,t+2} &\equiv u_{j,s,t+2}/u_{j,s,t+1} = \frac{\exp(V_{j,s,t+2})}{\exp(V_{j,s,t+1})} = \exp(V_{j,s,t+2} - V_{j,s,t+1}) \\ (\dot{u}_{j,s,t+2})^{\frac{\beta}{v}} &= \exp(V_{j,s,t+2} - V_{j,s,t+1})^{\frac{\beta}{v}}. \end{aligned}$$

Introducing this in the previous results and writing the equations for period t instead of $t+1$, we obtain

$$\mu_{ji,sk|i,t} = \frac{\mu_{ji,sk|i,t-1} \dot{u}_{i,k,t+1}^{\frac{\beta}{v}}}{\sum_{h=0}^S \mu_{ji,sh|i,t-1} \dot{u}_{i,h,t+1}^{\frac{\beta}{v}}} \quad (\text{B1})$$

$$\mu_{ji,s\#,t} = \frac{\mu_{ji,s\#,t-1} \left(\sum_{h=0}^S \mu_{ji,sh|i,t-1} \dot{u}_{i,h,t+1}^{\frac{\beta}{v}} \right)^{v/\kappa}}{\sum_{m=1}^I \mu_{jm,s\#,t-1} \left(\sum_{h=0}^S \mu_{jm,sh|m,t-1} \dot{u}_{m,h,t+1}^{\frac{\beta}{v}} \right)^{v/\kappa}}. \quad (\text{B2})$$

Take the difference between $V_{j,s,t+1}$ and $V_{j,s,t}$ using equation (8) to get

$$\begin{aligned} V_{j,s,t+1} - V_{j,s,t} &= U_{j,s,t+1} - U_{j,s,t} \\ &+ \ln \left(\frac{\left(\sum_{i=1}^I \left(\sum_{k=0}^S \exp(\beta V_{i,k,t+2} - \varphi_{ji,sk})^{1/v} \right)^{v/\kappa} \right)^\kappa}{\left(\sum_{m=1}^I \left(\sum_{h=0}^S \exp(\beta V_{m,h,t+1} - \varphi_{jm,sh})^{1/v} \right)^{v/\kappa} \right)^\kappa} \right) \\ e^{V_{j,s,t+1} - V_{j,s,t}} &= \exp(\ln((\omega_{j,s,t+1} \Delta_{j,s,t+1}) / (\omega_{j,s,t} \Delta_{j,s,t}))) \\ &\cdot \exp \left(\ln \left(\sum_{i=1}^I \left(\sum_{k=0}^S \dot{u}_{i,k,t+2}^{\frac{\beta}{v}} \mu_{ji,sk|i,t} \right)^{v/\kappa} \mu_{ji,s\#,t} \right)^\kappa \right). \end{aligned}$$

Thus, we finally obtain

$$\dot{u}_{j,s,t+1} = \dot{\omega}_{j,s,t+1} \dot{\Delta}_{j,s,t+1} \left(\sum_{i=1}^I \mu_{ji,s\#,t} \left(\sum_{k=0}^S \mu_{ji,sk|i,t} \dot{u}_{i,k,t+2}^{\frac{\beta}{v}} \right)^{v/\kappa} \right)^\kappa. \quad (\text{B3})$$

The equilibrium in changes includes equations (B1), (B2), (B3), together with the dot versions of the remaining equations in (3) - (19).

B.4 Algorithm to Solve the Dot System

Group the equations of the dot equilibrium system into 3 categories:

1. The ones that are needed to obtain new migration and new labor supply from a guess of utilities (block 1):

$$\begin{aligned} \mu_{ji,sk|i,t} &= \frac{\mu_{ji,sk|i,t-1} \dot{u}_{i,k,t+1}^{\frac{\beta}{v}}}{\sum_{h=0}^S \mu_{ji,sh|i,t-1} \dot{u}_{i,h,t+1}^{\frac{\beta}{v}}} \\ \mu_{ji,s\#,t} &= \frac{\mu_{ji,s\#,t-1} \left(\sum_{h=0}^S \mu_{ji,sh|i,t-1} \dot{u}_{i,h,t+1}^{\frac{\beta}{v}} \right)^{v/\kappa}}{\sum_{m=1}^I \mu_{jm,s\#,t-1} \left(\sum_{h=0}^S \mu_{jm,sh|m,t-1} \dot{u}_{m,h,t+1}^{\frac{\beta}{v}} \right)^{v/\kappa}} \end{aligned}$$

$$\ell_{i,s,t} = \sum_{j=1}^I \sum_{k=0}^S \mu_{ji,ks|i,t-1} \mu_{ji,k\#,t-1} \ell_{j,k,t-1}$$

With these equations, if one has an initial distribution of labor supply ($\ell_{i,s,0}$), initial mobility matrices ($\mu_{ji,sk|i,0}$ and $\mu_{ji,s\#,0}$) and an initial guess for the utility dots ($\dot{u}_{i,s,t}^{(0)} \forall t$), one can obtain the entire path of labor supplies ($\ell_{i,s,t} \forall t > 0$), and the entire path of mobility matrices ($\mu_{ji,sk|i,t}$ and $\mu_{ji,s\#,t} \forall t > 0$) without needing to use the other equations at all.

2. The ones that are needed to obtain the temporary equilibrium (wages, actual labor, sectoral prices, trade shares, revenue levels) from a given set of shocks and labor supply (block 2):

$$\begin{aligned} \dot{p}_{i,s,t}^{1-\sigma_s} &= \sum_{j=1}^I \lambda_{ji,s,t-1} \left(\dot{\tau}_{ij,s,t} \dot{A}_{j,s,t}^{-1} \dot{W}_{j,s,t}^{\phi_{i,s}} \prod_{k=1}^S \dot{p}_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s} \\ \lambda_{ij,s,t} &= \frac{\lambda_{ij,s,t-1} (\dot{\tau}_{ij,s,t} \dot{A}_{i,s,t}^{-1} \dot{W}_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S \dot{p}_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\sum_{r=1}^I \lambda_{rj,s,t-1} (\dot{\tau}_{rj,s,t} \dot{A}_{r,s,t}^{-1} \dot{W}_{r,s,t}^{\phi_{r,s}} \prod_{k=1}^S \dot{p}_{r,k,t}^{\phi_{r,ks}})^{1-\sigma_s}} \\ R_{i,s,t} &= \sum_{j=1}^I \lambda_{ij,s,t} \left(\alpha_{j,s} \left(\sum_s \dot{W}_{j,s,t} \dot{L}_{j,s,t} Y_{j,s,t-1} + D_{j,t} \right) + \sum_{k=1}^S \phi_{j,sk} R_{j,k,t} \right) \\ \dot{W}_{i,s,t} \dot{L}_{i,s,t} Y_{i,s,t-1} &= \phi_{i,s} R_{i,s,t} \\ \prod_{q=1}^t \dot{L}_{i,s,q} &\leq \prod_{q=1}^t \dot{\ell}_{i,s,q}, \dot{W}_{i,s,t} \geq \delta_{i,s}, \text{ Complementary Slackness} \\ \gamma \sum_{i=1}^I \sum_{s=1}^S Y_{i,s,t-1} &= \sum_{i=1}^I \sum_{s=1}^S \dot{W}_{i,s,t} \dot{L}_{i,s,t} Y_{i,s,t-1} \end{aligned}$$

3. The ones that are needed to update the guess for the path of utilities (block 3):

$$\begin{aligned} \dot{p}_{i,t} &= \prod_{s=1}^S \dot{p}_{i,s,t}^{\alpha_{i,s}} \\ \dot{\omega}_{i,s,t} &= \frac{\dot{W}_{i,s,t} \dot{L}_{i,s,t}}{\dot{p}_{i,t} \dot{\ell}_{i,s,t}} \text{ (but with } \dot{\omega}_{i,s,t} = 1 \text{ if } s = 0) \\ \dot{u}_{j,s,t+1} &= \dot{\omega}_{j,s,t+1} \dot{\Delta}_{j,s,t+1} \left(\sum_{i=1}^I \mu_{ji,s\#,t} \left(\sum_{k=0}^S \mu_{ji,sk|i,t} \dot{u}_{i,k,t+2}^{\frac{\beta}{v}} \right)^{v/\kappa} \right)^{\kappa} \end{aligned}$$

The algorithm works as follows:

1. Guess a path for the utility dots (which can be all of them being equal to one).
2. Use block one to obtain paths for the μ 's and ℓ 's using the guessed path for utility.
3. Use block two to solve the temporary equilibrium using the path for the ℓ 's.

4. Use block three to obtain a new guess for the utility dots. This uses the fact that in a far enough point in the future (called T) even the new guess of utility dots should have $\dot{u}_{i,s,T}^{(1)} = 1$. With $\dot{u}_{i,s,T}^{(1)} = 1$, the path for μ 's and the sectoral compensations one can obtain $\dot{u}_{i,s,T-1}^{(1)}$. And from those obtain $\dot{u}_{i,s,T-2}^{(1)}$, and so on until $\dot{u}_{i,s,1}^{(1)}$.
5. If the two guessed paths of utility dots $\dot{u}^{(0)}$ and $\dot{u}^{(1)}$ are close enough, stop the algorithm, otherwise return to item one with the new guess and iterate again.

B.5 Equilibrium in Counterfactual Relative to Baseline (Hats)

Now we want to describe the equilibrium equations in ratios of changes in a counterfactual economy relative to the same changes in the baseline economy. We will use the notation $\hat{x}_t = \dot{x}'_t / \dot{x}_t$, where \dot{x}'_t is the relative change from period $t-1$ to t in the counterfactual economy and \dot{x}_t is the same thing but for the baseline economy. First, we want to get the evolution of $\mu'_{ji,sk|i,t}$. Start from equation (B1) for the case of the counterfactual economy,

$$\mu'_{ji,sk|i,t} = \frac{\mu'_{ji,sk|i,t-1} (\dot{u}'_{i,k,t+1})^{\frac{\beta}{v}}}{\sum_{h=0}^S \mu'_{ji,sh|i,t-1} (\dot{u}'_{i,h,t+1})^{\frac{\beta}{v}}}.$$

Divide this by the same expression in the case of the baseline economy and rearrange to get:

$$\mu'_{ji,sk|i,t} = \frac{\mu'_{ji,sk|i,t-1} \dot{\mu}_{ji,sk|i,t} \hat{u}_{i,k,t+1}^{\frac{\beta}{v}}}{\sum_{h=0}^S \mu'_{ji,sh|i,t-1} \dot{\mu}_{ji,sh|i,t} \hat{u}_{i,h,t+1}^{\frac{\beta}{v}}}. \quad (\text{B4})$$

To obtain the evolution of $\mu'_{ji,s\#,t}$, start from equation (B2) for the counterfactual economy,

$$\mu'_{ji,s\#,t} = \frac{\mu'_{ji,s\#,t-1} \left(\sum_{h=0}^S \mu'_{ji,sh|i,t-1} (\dot{u}'_{i,h,t+1})^{\frac{\beta}{v}} \right)^{v/\kappa}}{\sum_{m=1}^I \mu'_{jm,s\#,t-1} \left(\sum_{h=0}^S \mu'_{jm,sh|m,t-1} (\dot{u}'_{m,h,t+1})^{\frac{\beta}{v}} \right)^{v/\kappa}}.$$

Divide this by the same expression in the case of the baseline economy and rearrange to get:

$$\mu'_{ji,s\#,t} = \frac{\mu'_{ji,s\#,t-1} \dot{\mu}_{ji,s\#,t} \left(\sum_{h=0}^S \mu'_{ji,sh|i,t-1} \dot{\mu}_{ji,sh|i,t} \hat{u}_{i,h,t+1}^{\frac{\beta}{v}} \right)^{v/\kappa}}{\sum_{m=1}^I \mu'_{jm,s\#,t-1} \dot{\mu}_{jm,s\#,t} \left(\sum_{h=0}^S \mu'_{jm,sh|m,t-1} \dot{\mu}_{jm,sh|m,t} \hat{u}_{m,h,t+1}^{\frac{\beta}{v}} \right)^{v/\kappa}}. \quad (\text{B5})$$

Now we want to derive an expression for utility in hats. Start from equation (B3) for the counterfactual economy (but for period t instead of $t + 1$):

$$\hat{u}'_{j,s,t} = \hat{\omega}'_{j,s,t} \hat{\Delta}'_{j,s,t} \left(\sum_{i=1}^I \mu'_{ji,s\#,t-1} \left(\sum_{k=0}^S \mu'_{ji,sk|i,t-1} (\hat{u}'_{i,k,t+1})^{\frac{\beta}{v}} \right)^{v/\kappa} \right)^{\kappa}.$$

Dividing by this equation in the baseline economy and rearranging yields

$$\hat{u}_{j,s,t} = \hat{\omega}_{j,s,t} \hat{\Delta}_{j,s,t} \left(\sum_{i=1}^I \mu'_{ji,s\#,t-1} \hat{\mu}_{ji,s\#,t} \left(\sum_{k=0}^S \mu'_{ji,sk|i,t-1} \hat{\mu}_{ji,sk|i,t} \hat{u}_{i,k,t+1}^{\frac{\beta}{v}} \right)^{v/\kappa} \right)^{\kappa}. \quad (\text{B6})$$

However, at $t = 1$ the equilibrium conditions are slightly different. This is the result of the timing assumption in CDP (which we adopt in this paper too), that the counterfactual fundamentals are unknown before $t = 1$. This means that at $t = 0$, $\hat{u}_{j,s,0} = 1$, $\mu'_{ji,sk|i,0} = \mu_{ji,sk|i,0}$, $\mu'_{ji,s\#,0} = \mu_{ji,s\#,0}$, and $\ell'_{i,k,1} = \ell_{i,k,1} = \sum_{j=1}^I \sum_{s=0}^S \mu_{ji,sk|i,0} \mu_{ji,s\#,0} \ell_{j,s,0}$. To account for the unexpected change in fundamentals at $t = 1$, the right equations are

$$\mu'_{ji,sk|i,1} = \frac{\theta_{ji,sk|i,0} \hat{u}_{i,k,2}^{\beta/v}}{\sum_{h=0}^S \theta_{ji,sh|i,0} \hat{u}_{i,h,2}^{\beta/v}} \quad (\text{B7})$$

$$\mu'_{ji,s\#,1} = \frac{\mu_{ji,s\#,1} \left(\sum_{h=0}^S \theta_{ji,sh|i,0} \hat{u}_{i,h,2}^{\beta/v} \right)^{v/\kappa}}{\sum_{m=1}^I \mu_{jm,s\#,1} \left(\sum_{h=0}^S \theta_{jm,sh|m,0} \hat{u}_{m,h,2}^{\beta/v} \right)^{v/\kappa}} \quad (\text{B8})$$

$$\hat{u}_{j,s,1} = \hat{\omega}_{j,s,1} \hat{\Delta}_{j,s,1} \left(\sum_{i=1}^I \mu_{ji,s\#,1} \left(\sum_{k=0}^S \theta_{ji,sk|i,0} \hat{u}_{i,k,2}^{\beta/v} \right)^{v/\kappa} \right)^{\kappa}, \quad (\text{B9})$$

where

$$\theta_{ji,sk|i,0} \equiv \mu_{ji,sk|i,1} \hat{u}_{i,k,1}^{\beta/v}.$$

The equilibrium in hats includes equations (B4), (B5), (B6), together with the hat versions of the remaining equations in (3) - (19).

B.6 Algorithm to Solve the Hat System

As in the previous algorithm, group the equations into 3 categories:

1. The ones that are needed to obtain new mobility shares and new labor supply from a guess of utilities (block 1):

$$\begin{aligned}\mu'_{ji,sk|i,t} &= \frac{\mu'_{ji,sk|i,t-1} \dot{\mu}_{ji,sk|i,t} \hat{u}_{i,k,t+1}^{\frac{\beta}{v}}}{\sum_{h=0}^S \mu'_{ji,sh|i,t-1} \dot{\mu}_{ji,sh|i,t} \hat{u}_{i,h,t+1}^{\frac{\beta}{v}}} \\ \mu'_{ji,s\#,t} &= \frac{\mu'_{ji,s\#,t-1} \dot{\mu}_{ji,s\#,t} \left(\sum_{h=0}^S \mu'_{ji,sh|i,t-1} \dot{\mu}_{ji,sh|i,t} \hat{u}_{i,h,t+1}^{\frac{\beta}{v}} \right)^{v/\kappa}}{\sum_{m=1}^I \mu'_{jm,s\#,t-1} \dot{\mu}_{jm,s\#,t} \left(\sum_{h=0}^S \mu'_{jm,sh|m,t-1} \dot{\mu}_{jm,sh|m,t} \hat{u}_{m,h,t+1}^{\frac{\beta}{v}} \right)^{v/\kappa}} \\ \ell'_{i,s,t} &= \sum_{j=1}^I \sum_{k=0}^S \mu'_{ji,ks|i,t-1} \mu'_{ji,k\#,t-1} \ell'_{j,k,t-1}\end{aligned}$$

But period one works differently:

$$\begin{aligned}\mu'_{ji,sk|i,1} &= \frac{\theta_{ji,sk|i,0} \hat{u}_{i,k,2}^{\beta/v}}{\sum_{h=0}^S \theta_{ji,sh|i,0} \hat{u}_{i,h,2}^{\beta/v}} \\ \mu'_{ji,s\#,1} &= \frac{\mu_{ji,s\#,1} \left(\sum_{h=0}^S \theta_{ji,sh|i,0} \hat{u}_{i,h,2}^{\beta/v} \right)^{v/\kappa}}{\sum_{m=1}^I \mu_{jm,s\#,1} \left(\sum_{h=0}^S \theta_{jm,sh|m,0} \hat{u}_{m,h,2}^{\beta/v} \right)^{v/\kappa}} \\ \theta_{ji,sk|i,0} &\equiv \mu_{ji,sk|i,1} \hat{u}_{i,k,1}^{\beta/v}\end{aligned}$$

With these equations, if one has an initial distribution of labor supply ($\ell'_{i,s,0}$, which should be the same as $\ell_{i,s,0}$), the mobility matrices in the baseline economy and an initial guess for the utility hats ($\hat{u}_{i,s,t}^{(0)} \forall t$), one can obtain the entire path of labor supplies ($\ell'_{i,s,t} \forall t > 0$), and the entire path of mobility matrices without needing to use the other equations at all.

2. The ones that are needed to obtain the temporary equilibrium (wages, actual labor, sectoral prices, trade shares, revenue levels) from a given set of shocks and labor supply (block 2):

$$\begin{aligned}\hat{p}_{i,s,t}^{1-\sigma_s} &= \sum_{j=1}^I \lambda'_{ji,s,t-1} \dot{\lambda}_{ji,s,t} \left(\hat{\tau}_{ji,s,t} \hat{A}_{j,s,t}^{-1} \hat{W}_{j,s,t}^{\phi_{j,s}} \prod_{k=1}^S \hat{p}_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s} \\ \lambda'_{ij,s,t} &= \frac{\lambda'_{ij,s,t-1} \dot{\lambda}_{ij,s,t} (\hat{\tau}_{ij,s,t} \hat{A}_{i,s,t}^{-1} \hat{W}_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S \hat{p}_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\hat{p}_{j,s,t}^{1-\sigma_s}} \\ R'_{i,s,t} &= \sum_{j=1}^I \lambda'_{ij,s,t} \left(\alpha_{j,s} \left(\sum_s \hat{W}_{j,s,t} \hat{L}_{j,s,t} Y'_{j,s,t-1} \hat{W}_{j,s,t} \hat{L}_{j,s,t} + D'_{j,t} \right) + \sum_{k=1}^S \phi_{j,sk} R'_{j,k,t} \right) \\ \phi_{i,s} R'_{i,s,t} &= \hat{W}_{i,s,t} \hat{L}_{i,s,t} Y'_{i,s,t-1} \hat{W}_{i,s,t} \hat{L}_{i,s,t} \\ \prod_{q=1}^t \hat{L}_{i,s,q} \hat{L}_{i,s,t} &\leq \prod_{q=1}^t \ell'_{i,s,q}, \quad \hat{W}_{i,s,t} \hat{W}_{i,s,t} \geq \delta_{i,s}, \quad \text{Complementary Slackness} \\ \sum_{i=1}^I \sum_{s=1}^S Y'_{i,s,t-1} &= \frac{1}{\gamma} \sum_{i=1}^I \sum_{s=1}^S \hat{W}_{i,s,t} \hat{L}_{i,s,t} Y'_{i,s,t-1} \hat{W}_{i,s,t} \hat{L}_{i,s,t}\end{aligned}$$

With these equations, if we have a set of shocks in hats ($\hat{\tau}$ and \hat{A} , as well as deficits in the counterfactual economy D'), together with initial values for the counterfactual economy (like trade shares and nominal incomes) and the solution for the baseline economy (including trade shares dot, wages dot and labor dot), we can solve for hat prices, new trade shares in levels, new revenues in levels, actual labor hats and wages hat.

3. The ones that are needed to update the guess for the path of utilities (block 3):

$$\begin{aligned}\hat{P}_{i,t} &= \prod_{s=1}^S \hat{P}_{i,s,t}^{\alpha_{i,s}} \\ \hat{\omega}_{i,s,t} &= \frac{\hat{W}_{i,s,t} \hat{L}_{i,s,t}}{\hat{P}_{i,t} \hat{\ell}_{i,s,t}} \quad (\text{but with } \hat{\omega}_{i,s,t} = 1 \text{ if } s = 0) \\ \hat{u}_{j,s,t} &= \hat{\omega}_{j,s,t} \hat{\Delta}_{j,s,t} \left(\sum_{i=1}^I \mu'_{ji,s\#,t-1} \hat{\mu}_{ji,s\#,t} \left(\sum_{k=0}^S \mu'_{ji,sk|i,t-1} \hat{\mu}_{ji,sk|i,t} \hat{u}_{i,k,t+1}^{\beta/v} \right)^{v/\kappa} \right)^{\kappa}\end{aligned}$$

But period one works differently:

$$\begin{aligned}\hat{u}_{j,s,1} &= \hat{\omega}_{j,s,1} \hat{\Delta}_{j,s,1} \left(\sum_{i=1}^I \mu_{ji,s\#,1} \left(\sum_{k=0}^S \theta_{ji,sk|i,0} \hat{u}_{i,k,2}^{\beta/v} \right)^{v/\kappa} \right)^{\kappa} \\ \theta_{ji,sk|i,0} &\equiv \mu_{ji,sk|i,1} \hat{u}_{i,k,1}^{\beta/v}.\end{aligned}$$

The algorithm works as follows:

1. Guess a path for the utility hats (which can be all of them being equal to one).
2. Use block one to obtain paths for the μ' and ℓ' using the guessed path for the utility hat and the solution for the baseline economy.
3. Use block two to solve the temporary equilibrium using the path for ℓ' , the hat shocks and the solution for the baseline economy.
4. Use block three to obtain a new guess for the utility hats. This uses the sectoral compensations obtained in the previous step and the fact that in a far enough point in the future (called T) the change in utility in the baseline economy should be the same as the change in utility in the counterfactual, so we should have $\hat{u}_{i,s,T}^{(1)} = 1$. With $\hat{u}_{i,s,T}^{(1)} = 1$, the path for the μ' and the sectoral compensations one can obtain $\hat{u}_{i,s,T-1}^{(1)}$. And from those obtain $\hat{u}_{i,s,T-2}^{(1)}$, and so on until $\hat{u}_{i,s,2}^{(1)}$. $\hat{u}_{i,s,1}^{(1)}$ needs to be obtained with a special equation.
5. If the two guessed paths of utility hats $\hat{u}^{(0)}$ and $\hat{u}^{(1)}$ are close enough, stop the algorithm, oth-

erwise return to item one with the new guess and iterate again.

B.7 Algorithm to Solve the Temporary Equilibrium

Block two of the previously described outer algorithms (which solve the equilibrium system in dots or hats) solves for the temporary equilibrium of the baseline or counterfactual economy. Given the presence of an inequality constraint due to DNWR, solving this temporary equilibrium is an unwieldy process that would be infeasible with any traditional solver. To overcome this limitation, we develop an augmented version of [Alvarez and Lucas \(2007\)](#) to be able to handle the existence of DNWR. This inner algorithm is very efficient and allows us to solve the temporary equilibrium of the full model with DNWR extremely fast (provided we use the nominal anchor described in equation 19). In this appendix, we describe this inner algorithm in the case of the hat system. The inner algorithm for the dot system is analogous.

Notice first that, if one knows a given period's wages in hats (as well as the solution for the baseline economy, the previous period's trade shares, and the shocks to trade costs and technology), it is possible to obtain the corresponding prices in hats from the equation:

$$\hat{P}_{i,s,t}^{1-\sigma_s} = \sum_{j=1}^I \lambda'_{ji,s,t-1} \hat{\lambda}_{ji,s,t} \left(\hat{\tau}_{ji,s,t} \hat{A}_{j,s,t}^{-1} \hat{W}_{j,s,t}^{\phi_{j,s}} \prod_{k=1}^S \hat{P}_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s},$$

using traditional contraction mapping algorithms. The new trade shares can then easily be obtained from the following equation,

$$\lambda'_{ij,s,t} = \frac{\lambda'_{ij,s,t-1} \hat{\lambda}_{ij,s,t} (\hat{\tau}_{ij,s,t} \hat{A}_{i,s,t}^{-1} \hat{W}_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S \hat{P}_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\hat{P}_{j,s,t}^{1-\sigma_s}}.$$

Knowing the previous elements, employment in hats, the previous periods output levels, and the shock to deficits, allows one to solve for revenues using the linear (albeit massive) system described by the following set of equations

$$R'_{i,s,t} = \sum_{j=1}^I \lambda'_{ij,s,t} \left(\alpha_{j,s} \left(\sum_s \hat{W}_{j,s,t} \hat{L}_{j,s,t} Y'_{j,s,t-1} \hat{W}_{j,s,t} \hat{L}_{j,s,t} + D'_{j,t} \right) + \sum_{k=1}^S \phi_{j,sk} R'_{j,k,t} \right).$$

The previous argument implies that we can write revenues in the counterfactual economy in a given period as a function of that same period's wages and employment hats, i.e. $R'_{i,s,t}(\hat{\mathbf{W}}, \hat{\mathbf{L}})$

(where the bold W and L stand for the vector of wages and employment hats in all the regions and sectors).

What remains is to show how to solve the following system in wages and employment hats for all regions and sectors:

$$\begin{aligned}\phi_{i,s}R'_{i,s,t}(\hat{\mathbf{W}}, \hat{\mathbf{L}}) &= \hat{W}_{i,s,t}\hat{L}_{i,s,t}Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t} \\ \hat{L}_{i,s,t} &\leq L_{i,s,t}^U, \hat{W}_{i,s,t} \geq W_{i,s,t}^L, \text{ Complementary Slackness (C.S.)} \\ \sum_{i=1}^I \sum_{s=1}^S Y'_{i,s,t-1} &= \sum_{i=1}^I \sum_{s=1}^S \hat{W}_{i,s,t}\hat{L}_{i,s,t}Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}.\end{aligned}$$

This is where we will use an augmented version of the [Alvarez and Lucas \(2007\)](#) algorithm that accounts for the presence of DNWR. Imagine that one has an initial guess for wages and employment in hats, denoted $\hat{W}_{i,s,t}^{(0)}$ and $\hat{L}_{i,s,t}^{(0)}$. We use an algorithm that updates this guess as follows:

$$\begin{aligned}\hat{W}_{i,s,t}^{(1)} &= \max \left\{ \frac{(1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{L_{i,s,t}^U}, W_{i,s,t}^L \right\} \\ \hat{L}_{i,s,t}^{(1)} &= \min \left\{ L_{i,s,t}^U, \frac{(1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{W_{i,s,t}^L} \right\}.\end{aligned}$$

These new guesses obviously satisfy $\hat{L}_{i,s,t}^{(1)} \leq L_{i,s,t}^U$ and $\hat{W}_{i,s,t}^{(1)} \geq W_{i,s,t}^L$. The new guesses also satisfy the C.S. condition. To see this, notice that it cannot happen that:

$$\begin{aligned}\hat{W}_{i,s,t}^{(1)} &= \frac{(1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{L_{i,s,t}^U} \\ \hat{L}_{i,s,t}^{(1)} &= \frac{(1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{W_{i,s,t}^L},\end{aligned}$$

since that would require:

$$\begin{aligned}\frac{(1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{L_{i,s,t}^U} &\geq W_{i,s,t}^L \\ L_{i,s,t}^U &\geq \frac{(1-\lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{W_{i,s,t}^L}.\end{aligned}$$

Putting the last two inequalities together we get:

$$(1 - \lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}} \geq W_{i,s,t}^L L_{i,s,t}^U \geq (1 - \lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}(\hat{\mathbf{W}}^{(0)}, \hat{\mathbf{L}}^{(0)})}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}},$$

which is impossible unless both inequalities hold with equality (in which case all the relevant conditions are satisfied anyway). This means that unless we are in a knife edge case (where everything works fine) we are going to be either in the point:

$$\left(\hat{L}_{i,s,t}^{(1)}, \hat{W}_{i,s,t}^{(1)}\right) = \left(L_{i,s,t}^U, \frac{(1 - \lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{L_{i,s,t}^U}\right),$$

or in the point:

$$\left(\hat{L}_{i,s,t}^{(1)}, \hat{W}_{i,s,t}^{(1)}\right) = \left(\frac{(1 - \lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}}{W_{i,s,t}^L}, W_{i,s,t}^L\right),$$

which means that the C.S. condition is satisfied. It is also true that the new guess satisfies the nominal anchor if the previous guess did. To see this, notice that (from the observation that we are always in either of those special points) the following always holds:

$$\hat{W}_{i,s,t}^{(1)}\hat{L}_{i,s,t}^{(1)} = (1 - \lambda)\hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} + \lambda \frac{\phi_{i,s}R'_{i,s,t}}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}}.$$

Multiplying this by $Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}$ and summing it over i and s we get:

$$\begin{aligned} \sum_{i=1}^I \sum_{s=1}^S \hat{W}_{i,s,t}^{(1)}\hat{L}_{i,s,t}^{(1)} Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t} &= (1 - \lambda) \sum_{i=1}^I \sum_{s=1}^S \hat{W}_{i,s,t}^{(0)}\hat{L}_{i,s,t}^{(0)} Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t} \\ &+ \lambda \sum_{i=1}^I \sum_{s=1}^S \frac{\phi_{i,s}R'_{i,s,t}}{Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}} Y'_{i,s,t-1}\dot{W}_{i,s,t}\dot{L}_{i,s,t}. \end{aligned}$$

Focusing on the last term, it is possible to show that:

$$\sum_{i=1}^I \sum_{s=1}^S \phi_{i,s}R'_{i,s,t} = \sum_{j=1}^I \sum_{r=1}^S \hat{W}_{j,r,t}\hat{L}_{j,r,t} Y'_{j,r,t-1}\dot{W}_{j,r,t}\dot{L}_{j,r,t}.$$

This makes it clear that:

$$\sum_{i=1}^I \sum_{s=1}^S \hat{W}_{i,s,t}^{(1)} \hat{L}_{i,s,t}^{(1)} Y'_{i,s,t-1} \dot{W}_{i,s,t} \dot{L}_{i,s,t} = \sum_{i=1}^I \sum_{s=1}^S \hat{W}_{i,s,t}^{(0)} \hat{L}_{i,s,t}^{(0)} Y'_{i,s,t-1} \dot{W}_{i,s,t} \dot{L}_{i,s,t}.$$

Therefore, if the initial guess satisfies the nominal anchor the new guess will do so as well. Finally, when the algorithm converges, for example at iteration N , the following holds:

$$\hat{W}_{i,s,t}^{(N)} \hat{L}_{i,s,t}^{(N)} = (1 - \lambda) \hat{W}_{i,s,t}^{(N)} \hat{L}_{i,s,t}^{(N)} + \lambda \frac{\phi_{i,s} R'_{i,s,t}}{Y'_{i,s,t-1} \dot{W}_{i,s,t} \dot{L}_{i,s,t}},$$

which implies $\hat{W}_{i,s,t}^{(N)} \hat{L}_{i,s,t}^{(N)} Y'_{i,s,t-1} \dot{W}_{i,s,t} \dot{L}_{i,s,t} = \phi_{i,s} R'_{i,s,t}$, indicating that the final guess solves our desired system. We use the following initial guess which satisfies the nominal anchor,

$$\hat{W}_{i,s,t}^{(0)} = \frac{1}{\dot{W}_{i,s,t}}, \quad \hat{L}_{i,s,t}^{(0)} = \frac{1}{\dot{L}_{i,s,t}}.$$

B.8 Welfare

We start from our previous result that

$$V_{j,s,t} = \ln(\omega_{j,s,t} \Delta_{j,s,t}) + \kappa \ln \left(\sum_{i=1}^I \left(\sum_{k=0}^S \exp(\beta V_{i,k,t+1} - \varphi_{ji,sk})^{1/v} \right)^{v/\kappa} \right) + \gamma \kappa.$$

Using

$$\mu_{jj,s\#,t} = \frac{\left(\sum_{h=0}^S \exp(\beta V_{j,h,t+1} - \varphi_{jj,sh})^{1/v} \right)^{v/\kappa}}{\sum_{m=1}^I \left(\sum_{h=0}^S \exp(\beta V_{m,h,t+1} - \varphi_{jm,sh})^{1/v} \right)^{v/\kappa}}$$

we then have

$$\sum_{m=1}^I \left(\sum_{h=0}^S \exp(\beta V_{m,h,t+1} - \varphi_{jm,sh})^{1/v} \right)^{v/\kappa} = \mu_{jj,s\#,t}^{-1} \left(\sum_{h=0}^S \exp(\beta V_{j,h,t+1} - \varphi_{jj,sh})^{1/v} \right)^{v/\kappa}.$$

Next, using

$$\mu_{jj,ss|j,t} = \frac{\exp(\beta V_{j,s,t+1})^{1/v}}{\sum_{h=0}^S \exp(\beta V_{j,h,t+1} - \varphi_{jj,sh})^{1/v}},$$

we have

$$\sum_{h=0}^S \exp(\beta V_{j,h,t+1} - \varphi_{jj,sh})^{1/v} = \mu_{jj,ss|j,t}^{-1} \exp(\beta V_{j,s,t+1})^{1/v},$$

and hence

$$\mu_{jj,s\#,t}^{-1} \left(\sum_{h=0}^S \exp(\beta V_{j,h,t+1} - \varphi_{jj,sh})^{1/\nu} \right)^{\nu/\kappa} = \mu_{jj,s\#,t}^{-1} \mu_{jj,ss|j,t}^{-\nu/\kappa} \exp(\beta V_{j,s,t+1})^{1/\kappa}.$$

This implies that

$$\kappa \ln \left(\sum_{m=1}^I \left(\sum_{h=0}^S \exp(\beta V_{m,h,t+1} - \varphi_{jm,sh})^{1/\nu} \right)^{\nu/\kappa} \right) = \beta V_{j,s,t+1} - \kappa \ln(\mu_{jj,s\#,t}) - \nu \ln(\mu_{jj,ss|j,t}).$$

We then write

$$V_{j,s,t} = \ln(\omega_{j,s,t} \Delta_{j,s,t}) - \kappa \ln(\mu_{jj,s\#,t}) - \nu \ln(\mu_{jj,ss|j,t}) + \gamma\kappa + \beta V_{j,s,t+1}.$$

Iterating this equation forward, we obtain

$$V_{j,s,t} = \sum_{r=t}^{\infty} \beta^{r-t} \left(\ln(\omega_{j,s,r} \Delta_{j,s,r}) - \kappa \ln(\mu_{jj,s\#,r}) - \nu \ln(\mu_{jj,ss|j,r}) + \gamma\kappa \right).$$

We define the EV in consumption for market js at time $t = 0$ to be the scalar $\zeta_{j,s}$ such that

$$V'_{j,s,0} = V_{j,s,0} + \sum_{r=0}^{\infty} \beta^r \ln(\zeta_{j,s}) = \sum_{r=0}^{\infty} \beta^r \left(\ln \left(\frac{\omega_{j,s,r} \Delta_{j,s,r} \zeta_{j,s}}{(\mu_{jj,ss|j,r})^\nu (\mu_{jj,s\#,r})^\kappa} \right) + \gamma\kappa \right).$$

Defining $\mathcal{V}_{j,s} \equiv \ln(\zeta_{j,s})$ and rearranging the previous definition, we can write:

$$\begin{aligned} (V'_{j,s,0} - V_{j,s,0}) &= \mathcal{V}_{j,s} \sum_{r=0}^{\infty} \beta^r \\ \mathcal{V}_{j,s} &= (1 - \beta)(V'_{j,s,0} - V_{j,s,0}) \\ &= \sum_{r=1}^{\infty} \beta^r \ln \left(\frac{\hat{\omega}_{j,s,r} \hat{\Delta}_{j,s,r}}{(\hat{\mu}_{jj,ss|j,r})^\nu (\hat{\mu}_{jj,s\#,r})^\kappa} \right), \end{aligned}$$

which is the expression that we will use for the “welfare change” stemming from the China shock, formally the equivalent variation change in consumption due to the shock. Notice that, since the welfare effects from the China shock will be small in percentage terms, then $\mathcal{V}_{j,s} = \ln(\zeta_{j,s}) \approx \zeta_{j,s} - 1$.

B.9 More on Calibration

As discussed in the main text, the multiplicative nature of our productivity decomposition, $\hat{A}_{China,s,t} = \hat{A}_{China,t}^1 \hat{A}_{China,s}^2$, implies that their level is not identified. For example, if we multiply all the $\hat{A}_{China,s}^2$ by a constant c and we divide all the $\hat{A}_{China,t}^1$ by c , then we would have the same $\hat{A}_{China,s,t}$. Thus, we use the normalization $\sum_{s=1}^S \hat{A}_{China,s}^2 = 1$. Correspondingly, the model is only able to produce changes in imports that satisfy $\sum_{t=2001}^{end} \Delta X_{C,US,t}^{model} = \sum_{s=1}^S \Delta X_{C,US,s}^{end-2000,model}$. This condition is automatically satisfied by the actual changes, i.e. $\sum_{t=2001}^{end} \Delta X_{C,US,t} = \sum_{s=1}^S \Delta X_{C,US,s}^{end-2000}$, but not necessarily by the predicted changes, due to the lack of a constant in the second regression. We adjust the predicted changes in manufacturing so that they satisfy: $\sum_{t=2001}^{end} \widehat{\Delta X_{C,US,t}} = \sum_{s=1}^S \widehat{\Delta X_{C,US,s}^{end-2000}}$, this adjustment is very small. In all of our applications we match our targets with an accuracy greater than 99.9%.

C Data Construction

In this appendix section, we provide details on the construction of the data we briefly described in Section 4. We divide this appendix into three parts. Appendix C.1 describes all data sources. Appendix C.2 discusses how we combine the different data sources to compute an internally consistent bilateral trade-flow matrix for all sectors for the years when all the data is available. It also discusses how we use the previous step to project bilateral trade flows between states and countries for the years before full data availability. Finally, Appendix C.3 discusses the construction of the initial employment allocations for all regions and the bilateral migration flows between sectors and U.S. states.

C.1 Data Description and Sources

List of sectors. We use a total of 14 market sectors (plus a home production sector). The list of market sectors includes 12 manufacturing sectors, one catch-all services sector, and one agriculture sector. We follow CDP in the selection of the 12 manufacturing sectors. These are: **1)** Food, beverage, and tobacco products (NAICS 311-312, WIOD sector 3); **2)** Textile, textile product mills, apparel, leather, and allied products (NAICS 313-316, WIOD sectors 4-5); **3)** Wood products, paper, printing, and related support activities (NAICS 321-323, WIOD sectors 6-7); **4)** Mining, petroleum and coal products (NAICS 211-213, 324, WIOD sectors 2, 8); **5)** Chemical (NAICS 325,

WIOD sector 9); **6**) Plastics and rubber products (NAICS 326, WIOD sector 10); **7**) Nonmetallic mineral products (NAICS 327, WIOD sector 11); **8**) Primary metal and fabricated metal products (NAICS 331-332, WIOD sector 12); **9**) Machinery (NAICS 333, WIOD sector 13); **10**) Computer and electronic products, and electrical equipment and appliance (NAICS 334-335, WIOD sector 14); **11**) Transportation equipment (NAICS 336, WIOD sector 15); **12**) Furniture and related products, and miscellaneous manufacturing (NAICS 337- 339, WIOD sector 16). There is a **13**) Services sector which includes Construction (NAICS 23, WIOD sector 18); Wholesale and retail trade sectors (NAICS 42-45, WIOD sectors 19-21); Accommodation and Food Services (NAICS 721-722, WIOD sector 22); transport services (NAICS 481-488, WIOD sectors 23-26); Information Services (NAICS 511-518, WIOD sector 27); Finance and Insurance (NAICS 521-525, WIOD sector 28); Real Estate (NAICS 531-533, WIOD sectors 29-30); Education (NAICS 61, WIOD sector 32); Health Care (NAICS 621-624, WIOD sector 33); and Other Services (NAICS 493, 541, 55, 561, 562, 711-713, 811-814, WIOD sector 34).¹

List of countries: We use data for 50 U.S. states, and 37 other countries, including an aggregated rest of the world. The list of countries is: Australia, Austria, Belgium, Bulgaria, Brazil, Canada, China, Cyprus, the Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Italy, Ireland, Japan, Lithuania, Mexico, the Netherlands, Poland, Portugal, Romania, Russia, Spain, the Slovak Republic, Slovenia, S. Korea, Sweden, Taiwan, Turkey, the United Kingdom, and the rest of the world.

Data on bilateral trade between countries. World Input-Output Database (WIOD). Release of 2013. We use data for 2000-2007 in our baseline and 2000-2011 in one of our extensions. We map the sectors in the WIOD database to our 14 market sectors in the following way: **1**) Food Products, Beverage, and Tobacco Products (c3); **2**) Textile, Textile Product Mills, Apparel, Leather, and Allied Products (c4-c5); **3**) Wood Products, Paper, Printing, and Related Support Activities (c6-c7); **4**) Petroleum and Coal Products (c8); **5**) Chemical (c9); **6**) Plastics and Rubber Products (c10); **7**) Nonmetallic Mineral Products (c11); **8**) Primary Metal and Fabricated Metal Products (c12); **9**) Machinery (c13); **10**) Computer and Electronic Products, and Electrical Equipment and Appliances (c14); **11**) Transportation Equipment (c15); **12**) Furniture and Related Products, and Miscellaneous Manufacturing (c16); **13**) Construction (c18), Wholesale and Retail Trade (c19-c21), Transport Services (c23-c26), Information Services (c27), Finance and Insurance (c28), Real Estate

¹The only difference with respect to CDP in the definition of manufacturing sectors is that we include Mining (NAICS 211-213) together with Petroleum and Coal Products (NAICS 324) in our sector 4.

(c29- c30); Education (c32); Health Care (c33), Accommodation and Food Services (c22), and Other Services (c34); **14** Agriculture and Mining (c1-c2). We follow [Costinot and Rodriguez-Clare \(2014\)](#) to remove the negative values in the trade data from WIOD.

Data on bilateral trade in manufacturing between U.S states. We combine the 2002 and 2007 Commodity Flow Survey (CFS) with the WIOD database. The CFS records shipments between U.S. states for 43 commodities classified according to the Standard Classification of Transported Goods (SCTG). We follow CDP and use CFS 2007 tables that cross-tabulate establishments by their assigned NAICS codes against commodities (SCTG) ship-ped by establishments within each of the NAICS codes. These tables allow for the mapping of SCTG to NAICS.

Data on bilateral trade in manufacturing and agriculture between U.S states and the rest of the countries. We obtain sector-level imports and exports between the 50 U.S. states and the list of other countries from the Import and Export Merchandise Trade Statistics, which is compiled by the U.S. Census Bureau. This dataset reports imports and exports in each NAICS sector between each U.S. state and each other country in the world.

Data on sectoral and regional value-added share in gross output. Value added for each of the 50 U.S. states and 14 sectors can be obtained from the BEA by subtracting taxes and subsidies from GDP data. In the cases when gross output was smaller than value added we constrain value added to equal gross output. For the list of countries, we obtain the share of value added in gross output from WIOD.

Data on services expenditure and production. We compute bilateral trade in services using a gravity approach explained in Appendix C.2. As part of these calculations, we require data on production and expenditure in services by region. We obtain U.S. state-level services GDP from the Regional Economic Accounts of the BEA. We obtain U.S. state-level services expenditure from the Personal Consumption Expenditures (PCE) database of BEA. Finally, for the list of other countries we compute total production and expenditure in services from WIOD.

Data on agriculture expenditure and production. We also compute bilateral trade in agriculture using a gravity approach explained in Appendix C.2. To get production in agriculture for the U.S. states, we combine the 2002 and 2007 Agriculture Census with the National Marine Fisheries Service Census to get state-level production data on crops, livestock and seafood. We infer state-level expenditure in agriculture from our gravity approach explained in Appendix C.2. Finally, for the list of other countries we compute total production and expenditure in agriculture from WIOD.

Data on population and geographic coordinates. As part of the gravity approach to compute bilateral trade in services, we also need to compute bilateral distances between regions. We follow the procedure used in the GeoDist dataset of CEPII to calculate international (and intranational) bilateral trade distances. We thus require data on the most populated cities in each country, the cities' coordinates and population, and each country's population. We obtained this information from the United Nations Population Division website. In particular, we use the population of urban agglomerations with 300,000 inhabitants or more in 2018, by country, for 2000-2007. For Austria, Cyprus, Denmark, Estonia, Hungary, Ireland, Lithuania, Slovakia and Slovenia we use the two most populated cities.² For the case of U.S. states, we use population and coordinates data for each U.S. county within each U.S. state. The data for the U.S. counties comes from the U.S. CENSUS.

Data on employment and migration flows. For the case of countries, we take data on employment by country and sector from the WIOD Socio Economic Accounts (WIOD-SEA). For the case of U.S. states, we take sector-level employment (including unemployment and non-participation) from the 5% sample PUMS files of the 2000 Census. We only keep observations with ages between 25 and 65, who are either employed, unemployed, or out of the labor force. We construct a matrix of migration flows between sectors and U.S. states by combining data from the American Community Survey (ACS) and the Current Population Survey (CPS). Finally, we abstract from international migration.

C.2 Construction of the Bilateral Trade Flows Between Regions

We follow the notation from [Costinot and Rodriguez-Clare \(2014\)](#) and omit the time subscripts t that are relevant in our quantitative model. Define $X_{ij,ks}$ as sales of intermediate goods from sector k in region i to sector s in region j , and $X_{ij,kF}$ as the sales of sector k in region i to the final consumer of region j . Our final objective is to construct a bilateral trade flows matrix between all regions in our sample with elements equal to $X_{ij,k} = \sum_s X_{ij,ks} + X_{ij,kF}$. This matrix allows us to compute the trade shares $\lambda_{ij,k}$, and the sector-level revenues $R_{j,k} = \sum_l X_{jl,k}$ for each region, which are crucial elements in our hat algebra described in Section 3.6.

As additional definitions, take $E_{j,k} = \sum_i X_{ij,k}$ as the total expenditure of region j in sector k , $F_{j,k} = \sum_i X_{ij,kF}$ as the final consumption in region j of sector k , $F_j = \sum_k F_{j,k}$ as the total final

²For the specific case of Cyprus, the cities' information comes from the country's Statistical Service.

consumption of region j , and $X_{j,ks} = \sum_i X_{ij,ks}$ as the total purchases that sector s in region j makes from sector k . We construct $X_{ij,k}$ in the four steps explained below. With some abuse of notation, we refer to a region i as a U.S. state (country) by $i \in US$ ($i \notin US$).

Step 1: Bilateral trade between countries. In the first step we focus on the case where both i and j are countries. Thus, we simply take $X_{ij,k} = X_{ij,k}^{WIOD}$, where $X_{ij,k}^{WIOD}$ are the bilateral trade flows that come directly from the WIOD database.

Step 2: Manufacturing trade among U.S. states. In the second step we focus on manufacturing bilateral trade between U.S. States. For this, we combine the closest Commodity Flow Survey (CFS) for each year with WIOD Data for the total trade of the U.S. with itself. We first compute the shares that each state i exports to state j in sector k represent in the total trade of sector k according to CFS. Then, we calculate the total exports of state i to state j in sector k as WIOD's U.S. trade with itself in sector k multiplied by the share computed in the previous step to ensure that bilateral trade between states adds up to the WIOD total.

Step 3: Manufacturing trade between U.S. states and countries. For the third step, we combine Census and WIOD data to calculate the trade flows between each of the 50 U.S. states and the other 37 country regions. There is limited availability for the state \times sector-level trade data coming from the CENSUS. Data for exports at the state \times sector-level starts in 2002 and data for imports starts in 2008. We scale state-level imports and exports data from the Import and Export Merchandise Trade Statistics to match the U.S. totals in WIOD. More precisely, the exports (imports) of state i to (from) country j in manufacturing sector k are computed as a proportion of WIOD's U.S. export (imports) to (from) country j in sector k . This proportion is equal to the exports (imports) of state i to (from) country j in sector k relative to the total U.S. exports (imports) to (from) country j in sector k .

Since the Import and Export Merchandise Trade Statistics data for exports starts in 2002 and for imports starts in 2008, the bilateral trade flows between regions for the years before the data starts cannot be computed directly from the data. We adapt our computation method to take into account this issue. All previous procedures remain the same. Denote $X_{ij,k}^{base}$ as the matrix $X_{ij,k}$ for the first year where the exports or imports data is available (the base year). Define the share of exports of U.S. State i in sector k , going to country j in the base year as $y_{ij,k}^{base} \equiv \frac{X_{ij,k}^{base}}{\sum_{h \in US} X_{hj,k}^{base}} \quad \forall i \in US, j \notin US$. Similarly, define the share of imports of U.S. state j in sector k , coming from country i in the base year as $e_{ij,k}^{base} \equiv \frac{X_{ij,k}^{base}}{\sum_{l \in US} X_{il,k}^{base}} \quad \forall i \notin US, j \in US$. Finally for each sector k in manufacturing or agriculture; and any year before the base year define $X_{ij,k} = e_{ij,k}^{base} X_{iUS,k}^{WIOD} \quad \forall i \notin US, \forall j \in US$

and $X_{ij,k} = y_{ij,k}^{base} X_{USj,k}^{WIOD} \quad \forall i \in US, \forall j \notin US.$

Step 4: Trade in services and trade in agriculture. We compute bilateral trade flows for services and agriculture separately using a gravity structure that matches WIOD totals for trade between countries (including the U.S.). We start with the standard gravity equation (for simplicity, we remove the subscript of the sector) $X_{ij} = \left(\frac{w_i \tau_{ij}}{P_j}\right)^{-\varepsilon} E_j$, where $P_j^{-\varepsilon} = \sum_i (w_i \tau_{ij})^{-\varepsilon}$. We know that $\sum_j X_{ij} = R_i$ and hence $\sum_j \left(\frac{w_i \tau_{ij}}{P_j}\right)^{-\varepsilon} E_j = R_i$. This implies $w_i^{-\varepsilon} \Pi_i^{-\varepsilon} = R_i$, where $\Pi_i^{-\varepsilon} = \sum_j \tau_{ij}^{-\varepsilon} P_j^\varepsilon E_j$. Let $\tilde{P}_j \equiv P_j^{-\varepsilon}$ and $\tilde{\Pi}_i \equiv \Pi_i^{-\varepsilon}$, and $\tilde{\tau}_{ij} \equiv \tau_{ij}^{-\varepsilon}$. Given $\{E_j\}$, $\{R_i\}$, and $\{\tilde{\tau}_{ij}\}$, one we can get $\{\tilde{P}_j\}$ and $\{\tilde{\Pi}_i\}$ from the following system:

$$\begin{aligned} \tilde{P}_j &= \sum_i \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i \\ \tilde{\Pi}_i &= \sum_j \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j \end{aligned} \quad (C1)$$

The solution for $\{\tilde{P}_j, \tilde{\Pi}_i\}$ is unique up to a constant (Fally, 2015). This indeterminacy requires a normalization. We thus impose $\tilde{P}_1 = 100$ in each exercise. Then one can compute our outcome of interest $\{X_{ij}\}$ from

$$X_{ij} = \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} \tilde{P}_j^{-1} R_i E_j. \quad (C2)$$

Computation of the bilateral resistance $\tilde{\tau}_{ij}$. To solve the gravity system, we must first compute $\tilde{\tau}_{ij} \forall i, j$. We proceed by assuming the following functional form: $\tilde{\tau}_{ij} = \beta_0^{l_{ij}} dist_{ij}^{\beta_1} \exp(\xi_{ij})$, where l_{ij} is an indicator variable equal to 1 if $i = j$, and ξ_{ij} is an idiosyncratic error term. β_1 captures the standard distance elasticity and β_0 captures the additional *inverse* resistance of trading with others versus with oneself.

To calculate $dist_{ij}$, we follow the same procedure used in the GeoDist dataset of CEPII to calculate international (and intranational) bilateral trade distances. The idea is to calculate the distance between two countries based on bilateral distances between the largest cities of those two countries, those inter-city distances being weighted by the share of the city in the overall country's population (Head and Mayer, 2002).

We use population for 2010 and coordinates data for all U.S. counties, and all cities around the world with more than 300,000 inhabitants. For those countries with less than two cities of this size, we take the two largest cities. Coordinates are important to calculate the physical bilateral distances in kms between each county r in state i and county s in state j ($d_{rs} \forall r \in i, s \in j$).

j and $\forall i, j = 1, \dots, 50$), and define $dist(ij)$ as:

$$dist(ij) = \left(\sum_{r \in i} \sum_{s \in j} \left(\frac{pop_r}{pop_i} \right) \left(\frac{pop_s}{pop_j} \right) d_{rs}^\theta \right)^{1/\theta}, \quad (C3)$$

where pop_h is the population of country/state h . We set $\theta = -1$.

Given our definition of $\tilde{\tau}_{ij}$ we can write the gravity equation between countries as $X_{ij} = \beta_0^{l_{ij}} dist_{ij}^{\beta_1} \exp(\xi_{ij}) \tilde{\Pi}_i^{-1} \tilde{P}_j^{-1} R_i E_j$. Taking logs we can write the previous equation as:

$$\ln X_{ij} = \delta_i^o + \delta_j^d + \tilde{\beta}_0 l_{ij} + \beta_1 \ln dist_{ij} + \xi_{ij}, \quad (C4)$$

where $\tilde{\beta}_0 = \ln \beta_0$ and the δ s are fixed effects. We first estimate the equation above separately for services and agriculture using a 2000-2011 panel of bilateral trade flows between countries from WIOD. We present our OLS estimation results in Table C.1. Columns (1) and (2) refer to the estimated coefficients for the case of services and agriculture, respectively. Both regressions include year-by-origin and year-by-destination fixed effects. We take these estimates and compute the bilateral resistance term in each sector as $\hat{\tau}_{ij} = \exp(\hat{\beta}_0 l_{ij} + \hat{\beta}_1 \ln dist_{ij})$.

Trade in services. As inputs, we need total expenditures in services for each region (E_i), as well as total production in services (R_i). For the case of countries we take this directly from WIOD. For the case of U.S. states we take these variables from the Regional Economic Accounts of the Bureau

Table C.1: Estimation of Own-Country Dummy and Distance Elasticity

	(1)	(2)
Dep. Var.: $\ln X_{ij,t}$	Services	Agriculture
l_{ij}	7.357*** (0.126)	4.143*** (0.145)
$\ln dist_{ij}$	-0.376*** (0.037)	-1.745*** (0.020)
Year \times Orig.	Yes	Yes
Year \times Dest.	Yes	Yes
Observations	17,328	17,328
Adjusted R^2	0.66	0.76

Notes: This table displays the OLS estimates of specifications analogous to the one in equation (C4). The outcome variable $\ln X_{ij,t}$ is the log exports of country i sent to country j . The own-country dummy l_{ij} is defined as an indicator function equal to one whenever country i is the same as country j . Finally, $\ln dist_{ij}$ is the log distance between country i and country j . This variable is computed according to equation (C3). Robust standard errors are presented in parenthesis. *** denotes statistical significance at the 1%.

of Economic Analysis. We scale the state-level services production and expenditures so that they aggregate to the U.S. totals in WIOD.

We incorporate the information on bilateral trade in services between countries (including the U.S.) that comes from WIOD to the gravity system of equation (C1) by first writing the system as $\tilde{P}_j = \sum_{i \notin US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i + \sum_{i \in US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i$ and $\tilde{\Pi}_i = \sum_{j \notin US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j + \sum_{j \in US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j$. Then, we define $\tilde{\lambda}_j \equiv 1 - \frac{\sum_{i \notin US} X_{ij}}{E_j}$ for $j \notin US$ (the share of imports of region $j \notin US$ coming from the U.S.) and $\tilde{\lambda}_i^* \equiv 1 - \frac{\sum_{j \notin US} X_{ij}}{R_i}$ for $i \notin US$ (total exports of region $i \notin US$ to other regions not in the U.S.). Using these two definitions and substituting $\tilde{\tau}_{ij} = X_{ij} \tilde{\Pi}_i \tilde{P}_j R_i^{-1} E_j^{-1}$ whenever $i, j \notin US$ in the previous system of equations we have the final system we solve for services:

$$\begin{aligned} \tilde{P}_j &= \sum_i \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i \quad j \in US \\ \tilde{\Pi}_i &= \sum_j \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j \quad i \in US \\ \tilde{\lambda}_j \tilde{P}_j &= \sum_{i \in US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i \quad j \notin US \\ \tilde{\lambda}_i^* \tilde{\Pi}_i &= \sum_{j \in US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j \quad i \notin US \end{aligned}$$

Once we find solutions for $\{\tilde{P}_j, \tilde{\Pi}_i\}$, we compute the final bilateral trade matrix according to equation (C2).

Trade in agriculture. As inputs, we need total expenditures in agriculture for each region (E_i), as well as total production in agriculture (R_i). For the case of countries we take this directly from WIOD. For the case of U.S. states we compute total production (R_i) by combining data from the Agriculture Census and the National Marine Fisheries Service Census. We scale the state-level agriculture production so that it aggregates to the U.S. total in WIOD. However, it is not possible to find state-level agriculture expenditure for U.S. states. To overcome this data unavailability, we combine the U.S. input-output matrix ($\phi_{j,ks}$) together with the shares of value-added in gross production ($\phi_{j,k}$) in order to compute a value of (E_i) that is consistent with the full bilateral trade matrix for all regions and all sectors.

In order to describe our procedure, note that the total expenditure of region j in sector k ($E_{j,k}$) could be written as $E_{j,k} = \sum_s \tilde{\phi}_{j,ks} R_{j,s} + F_{j,k}$, where $\tilde{\phi}_{j,ks} = \phi_{j,ks}(1 - \phi_{j,s})$. We make two assumptions. First, we assume that $\tilde{\phi}_{j,ks} = \tilde{\phi}_{US,ks} \forall j \in US$, which means that we assume common input-output matrix and value-added shares across U.S. states and equal to the ones of the U.S. as a whole. Second, we assume identical Cobb-Douglas preferences across U.S. states. This means that when

$j \in US$ we have that $F_{j,k} = \frac{F_j}{F_{US}} F_{US,k} = F_j \gamma_k$, where $\gamma_k \equiv \frac{F_{US,k}}{F_{US}}$. Using these two assumptions we get $F_j = E_{j,k} - \sum_s \tilde{\phi}_{j,ks} R_{j,s} + \sum_{r \neq k} (E_{j,r} - \sum_s \tilde{\phi}_{j,rs} R_{j,s})$. Substituting the previous equation in the definition of $E_{j,k}$ for the agriculture sector ($k = AG$), and $j \in US$ we find

$$E_{j,AG} = \sum_s \tilde{\phi}_{j,AGs} R_{j,s} + \frac{\gamma_{AG}}{1 - \gamma_{AG}} \sum_{r \neq AG} \left(E_{j,r} - \sum_s \tilde{\phi}_{j,rs} R_{j,s} \right),$$

which can be computed using state-level production of all sectors and state-level expenditure data of all other sectors (excluding agriculture), combined with the U.S.-level input-output matrix, value-added shares, and sector-level consumption shares.

Once we obtain the state-level expenditure values in agriculture, we can proceed with the gravity system in equation (C1). As in the case of services, we incorporate the information on bilateral trade in agriculture between countries that comes from WIOD. We also incorporate the bilateral trade in agriculture between U.S. states and other countries coming from the Import and Export Merchandise Trade Statistics. Thus, we only need to focus on $\{\tilde{P}_j\}_{j \in US}$ and $\{\tilde{\Pi}_i\}_{i \in US}$. Define $\chi_i^* = 1 - \sum_{j \notin US} \frac{X_{ij}}{R_i}$ for $i \in US$ (the share of sales of state i that stay in the U.S.) and $\chi_j = 1 - \sum_{i \notin US} \frac{X_{ij}}{E_{j,k}}$ for $j \in US$ (the share of purchases of state i that come from the U.S.). The final system we solve for agriculture becomes:

$$\begin{aligned} \chi_j \tilde{P}_j &= \sum_{i \in US} \tilde{\tau}_{ij} \tilde{\Pi}_i^{-1} R_i, \forall j \in US \\ \chi_i^* \tilde{\Pi}_i &= \sum_{j \in US} \tilde{\tau}_{ij} \tilde{P}_j^{-1} E_j, \forall i \in US \end{aligned}$$

As before, once we find solutions for $\{\tilde{P}_j, \tilde{\Pi}_i\}$, we compute the bilateral trade in agriculture between U.S. states according to equation (C2).

C.3 Initial Employment Allocations for each Region and Bilateral Migration Flows between Sectors and U.S. States

Employment allocation in each region and sector. For the case of countries outside of the U.S., we first compute the employment distribution by country-sector from the WIOD-SEA. We treat the unemployed and out-of-labor force as an additional sector. The data for that sector combines WIOD-SEA's worker population and each country's labor force participation rate from World Bank data. Since SEA does not include the RoW directly and since the remaining countries

in SEA are too few, we define RoW's employment such that its production to employment ratio equals the respective average ratio of the other 37 countries. This calculation is done separately for each sector.

For the case of U.S. states, we calculate the employment level for each state and sector (including unemployment and non-participation) in the year 2000 from the 5% sample PUMS files of the 2000 Census. We only keep observations type "P" (persons) aged 25 to 65, who are either employed or out of the labor force. Finally, we apply proportionality so that the aggregate employment at the sector level coincides with the totals for the U.S. in WIOD-SEA.

Workers' mobility matrix for U.S. states. Let $L_{ji,sk}$ be the number of workers who move from state j and sector s to state i and sector k between two periods (we ignore the time subscript for simplicity). We want to compute the mobility matrix for the shares $\mu_{ji,sk}$ for each origin state j , origin sector s , destination state i , and destination sector k , with the shares defined as $\mu_{ji,sk} = \frac{L_{ji,sk}}{\sum_{i'} \sum_{k'} L_{j i', s k'}}$. To do this, we combine data from the Current Population Survey (CPS), the American Community Survey (ACS), the IRS state-to-state migration data, and the sector-state employment data from BLS, as explained below.

The CPS provides details of people's employment status and industry each month, but it does not provide information regarding movements across states. This means that we can construct from the CPS data $L_{jj,sk}^{CPS} \quad \forall j \in \text{U.S. and any origin or destination sectors } s, k$ (intra-state flows of people between sectors). To remain internally consistent with the model, we only consider employed or out-of-the-labor force workers (i.e., we exclude unemployed workers).³

The ACS provides details of workers' current employment status, sector, and state. It also asks the state in which respondents lived the prior year. However, this survey does not provide information regarding people's employment status and sector in the previous year. This means that we can construct from the ACS data $L_{ji, \#k}^{ACS} \quad \forall j, i \in \text{U.S. and destination sector } k$ (interstate flows but without knowing the sector of origin, where the unknown component is labeled as #). Finally, the IRS state-to-state migration data allows us to construct the mobility between states regardless of their sector $L_{ji, \#\#}^{IRS} \quad \forall j, i \in \text{U.S.}$

³The CPS surveys households in a 4-8-4 format; that is, it interviews the household for four consecutive months, gives them an 8-month break, and interviews them again for four straight months. We start with the NBER version of the CPS. The first four monthly interviews are 12 months apart from the final four interviews, and the first four and final four are consecutive months. Since we are interested in recording annual changes, we only keep interview months (1,5) which is equivalent to following individuals for the first twelve months they appear in the survey. To avoid noise in our sample, we pooled observations for the previous two years and the following two years for the year of interest.

We combine these datasets to compute the labor transitions across states and sectors. We also apply proportionality to the flows from CPS and ACS to sum up the total flows of the IRS data (which do not require additional assumptions and are available for interstate movements). In particular, for movements between sectors within the same state we use the following rule:

$$L^{jj,sk} = L_{jj,##}^{IRS} \times \frac{L_{jj,sk}^{CPS}}{\sum_s \sum_k L_{jj,sk}^{CPS}} \quad \forall j, \forall s, k$$

For movements across states, we define:

$$L^{ji,sk} = \frac{L_{jj,sk}^{CPS}}{\sum_s L_{jj,sk}^{CPS}} \times L_{ji,##}^{IRS} \times \frac{\sum_i L_{ji,?k}^{ACS}}{\sum_i \sum_k L_{ji,?k}^{ACS}}$$

where, in the few cases when the diagonal value of the matrix (same state and sector in origin and destination) is zero, we change it to the minimum non-zero diagonal value.

Since the U.S. Census has the highest-quality data on the labor distribution by sector-state, we want our constructed flows to be consistent with sector-state data from the Census. However, this data is only available every ten years. Because of this, we first rely on sector-state data from the BLS (which is available yearly) to get sector-level employment for 1999 and 2000 and make our migration flows consistent with those employment vectors as explained below. One disadvantage of the BLS data is that it is based on a sample of the U.S. population and therefore its levels are less reliable than the ones from the Census (in particular, some sector-states have zero employment in both 1999 and 2000 in the BLS data but non-zero employment in the 2000 Census). To make the employment changes consistent with the 1999-2000 BLS data but the levels in 2000 consistent with the Census data, we construct the ratio of employment in 1999 to employment in 2000 for all sector-states using BLS data (after normalizing the total employment to be constant across time as it is in our model), winsorize these ratios at the 2.5% and 97.5% levels, and then multiply them by the 2000 Census data to generate a 1999 employment vector that is consistent with the levels of the 2000 Census but the changes in BLS, which we denote by “CBLS”.

Note that the migration shares imply that $L_{i,k,t+1} = \sum_{j=1}^I \sum_{s=0}^S \mu_{ji,sk} L_{j,s,t}$, where $L_{j,s,t}$ is the total employment in region j , sector s , at time t . Since we want the migration flows matrix for 1999-2000 to be consistent with the change in the stocks of workers across sector-state pairs that we observe in the data between 1999 and 2000 (which is much more reliable than the direct migration flows), we use the $\mu'_{ji,sk}$ s that are the closest to the ones constructed with the steps above but that satisfy that

$L_{i,k,2000}^{Census} = \sum_{j=1}^I \sum_{s=0}^S \mu'_{ji,sk} L_{j,s,1999}^{CBLS}$. Specifically, we minimize the sum of square differences between the new μ 's and the original ones subject to: (1) the new μ 's are consistent with the change in the stocks of workers across sector-states from the CBLS data, (2) they are greater than zero: $\mu'_{ji,sk} \geq 0$, and they sum to one for each sender market over all receiver markets: $\sum_{i=1}^I \sum_{k=0}^S \mu'_{ji,sk} = 1$, and (3) if the original μ matrix has a given flow as zero, then this must still be the case in the new μ' matrix: $\mu'_{ji,sk} = 0$ if $\mu_{ji,sk} = 0$. The change in the flows implied by this procedure is very small. In particular, the correlation between the original $\mu_{ji,sk}$ and the $\mu'_{ji,sk}$ is greater than 99.99%.

Mobility matrix for non-U.S. regions. We do not take the mobility matrix for each country outside of the U.S. from the data, which would be extremely cumbersome because we have 37 other countries. However, it can be shown (details provided upon request), that for a country with a single region (such as non-U.S. countries in our context), the fact that there are no mobility costs can be captured by setting a special mobility matrix between 1999 and 2000. Thus, we compute the elements of that mobility matrix between 1999 and 2000. To do this, we take as given the labor distribution in 1999 ($L_{i,s,0}$) and 2000 ($L_{i,s,1}$) and compute the following formula:

$$\mu_{ii,sk,0} = \frac{L_{i,k,1}}{\sum_{r=1}^S L_{i,r,0}}$$

Notice that the flows between sector s and sector k do not depend on information of the sender sector (s), which is implicitly encoding the information that in the countries outside of the U.S., mobility between sectors is frictionless.

C.4 Handling Negative Cobb-Douglas Shares

From our data on bilateral trade flows, labor shares, and input-output coefficients we can back out a set of implied values for the $\alpha_{j,s}$ Cobb-Douglas parameters. However, there may be situations where the implied α 's for a small fraction of the region-sectors are slightly negative, which is not consistent with our model. In this case, we modify the bilateral trade flow data to make it consistent with non-negative α 's. In order to do this, we first obtain the set of α 's implied by the original data, then we set the negative implied α 's to zero, and then we re-normalize the α 's so that they add up to one again in each region. This process yields a new set of non-negative alphas which we denote $\tilde{\alpha}_{j,s}$.

We then recover the bilateral trade flows that are compatible with the new $\tilde{\alpha}_{j,s}$. The equilibrium system to obtain these bilateral trade flows is a special case of the temporary equilibrium

“dot” system described in point (2) of Section B.4 for the year 2000, without DNWR, without mobility, without technology or trade shocks, and with $\gamma = 1$ in the nominal anchor:

$$\begin{aligned}
\dot{P}_{i,s,t}^{1-\sigma_s} &= \sum_{j=1}^I \lambda_{ji,s,t-1} \left(\dot{W}_{j,s,t}^{\phi_{j,s}} \prod_{k=1}^S \dot{P}_{j,k,t}^{\phi_{j,ks}} \right)^{1-\sigma_s} \\
\lambda_{ij,s,t} &= \frac{\lambda_{ij,s,t-1} (\dot{W}_{i,s,t}^{\phi_{i,s}} \prod_{k=1}^S \dot{P}_{i,k,t}^{\phi_{i,ks}})^{1-\sigma_s}}{\sum_{r=1}^I \lambda_{rj,s,t-1} (\dot{W}_{r,s,t}^{\phi_{r,s}} \prod_{k=1}^S \dot{P}_{r,k,t}^{\phi_{r,ks}})^{1-\sigma_s}} \\
R_{i,s,t} &= \sum_{j=1}^I \lambda_{ij,s,t} \left(\tilde{\alpha}_{j,s} \left(\sum_s \dot{W}_{j,s,t} Y_{j,s,t-1} + D_{j,t} \right) + \sum_{k=1}^S \phi_{j,sk} R_{j,k,t} \right) \\
\dot{W}_{i,s,t} Y_{i,s,t-1} &= \phi_{i,s} R_{i,s,t} \\
\sum_{i=1}^I \sum_{s=1}^S \dot{W}_{i,s,t} Y_{i,s,t-1} &= \sum_{i=1}^I \sum_{s=1}^S Y_{i,s,t-1}.
\end{aligned}$$

In this system, the λ_{t-1} , $\tilde{\alpha}_{j,s}$, Y_{t-1} , and D_t 's are all known, and the \dot{W} , \dot{P} , R_t and λ_t 's are the outcomes. From these outcomes we can construct the new bilateral trade flow matrix that is consistent with the non-negative $\tilde{\alpha}_{j,s}$. This process changes the original data by a negligible amount; the correlation between the constructed bilateral trade flows and the original ones is above 99.99%. A version of this process is also applied by CDP, who further equalize the α 's across regions.

D Exposure Measures

Consider an economy producing a set of homogeneous goods across sectors $s = 1, \dots, S$ with prices p_s . Labor is the only factor of production that is mobile across sectors, and there are decreasing returns to labor in each sector so that $q_s = F_s(l_s)$ with $F'_s(\cdot) > 0$ and $F''_s(\cdot) < 0$. Preferences are given by $U(c) - V(l)$, where $l \equiv \sum_s l_s$, $U(c)$ is homogeneous of degree one, and $V'(\cdot) > 0$ and $V''(\cdot) < 0$. We are interested in the effect of a foreign shock on employment in two different cases. In the first case the wage w is fixed and labor is fully determined by labor demand (we assume that labor supply is higher than labor demand at the fixed wage w), while in the second case the wage is fully flexible and clears the labor market. Below we show that further assuming that $\varepsilon(l_s) \equiv -\frac{F''_s(l_s)l_s}{F'_s(l_s)} = \varepsilon$ for all s and $\mu(l) \equiv \frac{V''(l)l}{V'(l)} = \mu$, then in the case of a fixed wage we have

$$d \ln l = \frac{1}{\varepsilon} \sum_s \left(\frac{p_s q_s}{I} \right) d \ln p_s \quad (\text{D1})$$

while in the case of flexible wages we have

$$d \ln l = \frac{1}{\varepsilon + \mu} \sum_s \left(\frac{p_s q_s - p_s c_s}{I} \right) d \ln p_s, \quad (\text{D2})$$

where $I \equiv \sum_s p_s q_s$. Thus, if the wage is fixed and if we know the log changes in prices resulting from the foreign shock then we can interact them with revenue shares, $\frac{p_s q_s}{I}$, to construct a Bartik-style sufficient statistic for the first order effect on employment. In contrast, if the wage fully adjusts to equalize labor supply and demand, then the appropriate weights (share components in the Bartik measure) for the price changes are instead given by net exports as a share of GDP, to capture the implied terms-of-trade effects. If the economy is small, then prices are exogenous and one could further replace $d \ln p_s$ by the underlying Chinese productivity shocks.

Let's start with the case where w is fixed. Fully differentiating the equilibrium condition $p_s F'_s(l_s) = w$ implies $d \ln l_s = \frac{d \ln p_s}{\varepsilon_s(l_s)}$, where $\varepsilon_s(l_s) \equiv -\frac{F''_s(l_s) l_s}{F'_s(l_s)}$. We then have $d \ln l = \sum_s m_s \frac{d \ln p_s}{\varepsilon_s(l_s)}$, where $m_s \equiv \frac{l_s}{\sum_s l_s}$. Assuming that $\varepsilon_s(l_s) = \varepsilon$ we know that $p_s q_s / I = m_s$ and hence we get (D1).

Now let's consider the case with a flexible wage. The equilibrium is given by w, l, λ and $\{l_s, c_s\}_s$ such that the following equations hold

$$p_s F'_s(l_s) = w \quad (\text{D3})$$

$$\frac{\partial U_s}{\partial c_s} = \lambda p_s \quad (\text{D4})$$

$$V'(l) = \lambda w \quad (\text{D5})$$

$$\sum_s l_s = l \quad (\text{D6})$$

$$\sum_s p_s c_s = \sum_s p_s f_s(l_s). \quad (\text{D7})$$

Differentiating equation (D5) yields $\mu(l) d \ln l = d \ln \lambda + d \ln w$, where $\mu(l) \equiv \frac{V''(l) l}{V'(l)}$. Thus

$$d \ln l = \frac{d \ln (w/P)}{\mu(l)}, \quad (\text{D8})$$

with $P \equiv 1/\lambda$. Next, totally differentiating equations (D3) and (D6) yields $d \ln p_s - \varepsilon d \ln l_s = d \ln w$ and $\sum_s m_s d \ln l_s = d \ln l$. Combined, the previous two equations imply $\sum m_s d \ln p_s - \varepsilon d \ln l = d \ln w$, which combined with (D8) implies (after some rearranging):

$$d \ln (w/P) = \frac{\mu}{\mu + \varepsilon} \left(\sum m_s d \ln p_s - d \ln P \right). \quad (\text{D9})$$

But equation (D4) implies that $\sum_s \frac{\partial U_s}{\partial c_s} c_s = \lambda \sum_s p_s c_s$. Since $U(c)$ is homogeneous of degree one this

implies $U(c) = \lambda \sum_s p_s c_s$. Totally differentiating this equation yields $\sum_s \frac{\partial U_s}{\partial c_s} dc_s = (\sum_s p_s c_s) d\lambda + \lambda \sum_s p_s dc_s + \lambda \sum_s c_s dp_s$. Using equation (D4) we get $\sum_s \lambda p_s dc_s = (\sum_s p_s c_s) d\lambda + \lambda \sum_s p_s dc_s + \lambda \sum_s c_s dp_s$, which, after simplifying, implies

$$d \ln P = d \ln (1/\lambda) = \sum_s \theta_s d \ln p_s, \quad (\text{D10})$$

where $\theta_s \equiv \frac{p_s c_s}{\sum_s p_s c_s}$. Plugging into (D9) and combining with (D8) we get

$$d \ln l = \frac{1}{\mu + \varepsilon} \sum (m_s - \theta_s) d \ln p_s = d \ln l.$$

Finally, note that $m_s \equiv \frac{l_s}{\sum_s l_s} = \frac{wl_s}{\sum_s wl_s} = \frac{p_s F'_s(l_s) l_s}{\sum_s p_s F'_s(l_s) l_s}$. Using $\varepsilon(l_s) \equiv -\frac{F''_s(l_s) l_s}{F'_s(l_s)} = \varepsilon$, we know that $F_s(l_s) \propto l_s^{1-\varepsilon}$ and $F'_s(l_s) \propto (1-\varepsilon) l_s^{-\varepsilon}$, hence $m_s = \frac{p_s F_s(l_s)}{\sum_s p_s F_s(l_s)} = \frac{p_s q_s}{\sum_s p_s q_s} = \frac{p_s q_s}{I}$. On the other hand, using (D7) we have $\theta_s \equiv \frac{p_s c_s}{\sum_s p_s c_s} = \frac{p_s c_s}{I}$. Combining all of this we obtain (D2).

D.1 “Horse Race” Between Different Exposure Measures

Table D.1: “Horse race” between different exposure measures in the baseline model with and without DNWR

	(1) Welf. Flex.	(2) Welf. DNWR	(3) Empl. Flex.	(4) Empl. DNWR
Constant	0.598** (0.056)	0.555** (0.071)	2.733** (0.313)	1.324* (0.512)
ADH Exposure	-0.027 (0.016)	-0.066** (0.020)	-0.079 (0.090)	-0.780** (0.147)
NX Exposure	-0.098** (0.016)	-0.110** (0.020)	-0.534** (0.090)	-0.617** (0.147)
N	50	50	50	50
R squared	0.500	0.516	0.462	0.557
Mean dep. var.	0.269	0.093	1.121	-2.350

Notes: This table shows the results of regressing several variables on a constant, ADH exposure, and net export exposure (renormalized to have the same mean and standard deviation as ADH exposure). The exposure variables are described in the text. The dependent variables are: welfare change from the China shock in the baseline model without DNWR (column 1), welfare change from the China shock in the baseline model with DNWR (column 2), percentage change in total employment between 2000 and 2007 in the baseline model without DNWR (column 3), and percentage change in total employment between 2000 and 2007 in the baseline model with DNWR (column 4). Stars denote significance, one for 5%, and two for 1%.

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