A Macroeconomic Model of Central Bank Digital Currency

Pascal Paul Mauricio Ulate Jing Cynthia Wu

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Abstract

We develop a quantitative DSGE model with monopolistic banks to study the macroeconomic effects of introducing a central bank digital currency (CBDC). Households benefit from an expansion of liquidity services and higher deposit rates as bank deposit market power is curtailed, while bank profits and lending decline. We quantify this trade-off across economies that differ in their level of interest rates. We find substantial welfare gains from introducing a CBDC with an optimal rate that can be approximated by a simple rule of thumb: the maximum between 0% and the policy rate minus 1%.

JEL codes: E3, E4, E5, G21, G51.
Keywords: Central bank digital currency, Banking, DSGE, Monetary policy.

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1 Introduction

The introduction of a retail central bank digital currency (CBDC), government-backed digital money accessible to the general public, is one of the most far-reaching innovations that central banks have considered in recent decades.[1] By late 2024, 134 countries and currency unions, accounting for 98% of global GDP, were exploring CBDCs.[2] This group includes most of the G20 economies, with particularly prominent initiatives such as the digital euro in the Euro Area. The launch of such a new form of currency could fundamentally reshape the financial landscape and raises a number of salient questions. First and foremost, is the introduction of a CBDC beneficial for an economy as a whole? Second, how should central banks set the interest rate on CBDCs, and how does this rate depend on the state of an economy, in particular the level of interest rates? And third, how does the presence of a CBDC affect the conduct of monetary policy and the behavior of an economy over the business cycle?

In this paper, we seek to answer these questions by proposing a new general equilibrium framework that features a realistic banking sector and that is closely calibrated to empirical evidence. At the heart of our model are monopolistic banks that have market power in deposit and loan markets. When a CBDC is introduced, households benefit from an expansion of liquidity services and higher deposit rates as bank deposit market power is curtailed. However, deposits also flow out of the banking system, the sector shrinks, and bank lending contracts. Central banks can balance these costs and benefits by steering CBDC demand with its interest rate, with the possibility to raise economy-wide welfare. For an economy matched to U.S. data, we find modest overall welfare gains for a CBDC that is introduced at zero interest or with its optimal rate of 0.8% per year. Larger welfare gains are feasible in economies with higher levels of interest rates if central banks set the rate on CBDC in such a way that it rises with the policy rate. In high interest rate environments, banks typically earn larger deposit spreads, and the benefit of reducing their deposit market power outweighs the other economic forces, leading to substantial overall welfare increases from CBDC introduction.

To gain intuition on the key mechanisms in the paper, we start by developing a static partial equilibrium model of deposit intermediation in Section 2. This simple framework has two important features. First, cash, deposits, and CBDC provide households with

[1] In contrast to retail CBDC, wholesale CBDC would only be available to financial and large nonfinancial institutions. Most central banks envision including a retail component in the CBDCs that they might eventually issue, as indicated in Figure 2a of the latest BIS annual survey on CBDC (Illes et al., 2025). Throughout the paper, we use the term CBDC as short for retail CBDC.

[2] See, e.g., the Atlantic Council's CBDC tracker: https://www.atlanticcouncil.org/cbdctracker/.

liquidity benefits, and the three instruments are imperfectly substitutable. Second, banks are monopolistic and set the deposit rate as a variable markdown on the policy rate. As a result, banks' deposit market power and the competition between the three liquidity-providing instruments jointly determine the difference between the policy rate and the deposit rate, which is the deposit spread that banks charge.

Our static framework illustrates the following features. In the absence of a CBDC, the deposit spread rises with the level of the policy rate, as banks gain market power relative to cash that pays zero interest (see, e.g., Drechsler et al., 2017). When a CBDC is introduced, the deposit spread decreases endogenously, as households value the liquidity benefits that a CBDC provides and lower their deposit holdings. The deposit spread falls the most if the CBDC rate is close to the policy rate, since a CBDC is a stronger competitor to deposits within that range. Thus, in an environment with a high policy rate and a large deposit spread, a CBDC that pays interest can be an important competitive force, lowering banks' deposit market power.

Besides the behavior of the deposit spread, the static framework also points to the key trade-off that determines the impact of introducing a CBDC on welfare in general equilibrium. On the one hand, households benefit from CBDC since it is a liquidity-providing instrument that they value and because it provides competition to bank deposits, which lowers deposit spreads. On the other hand, banks not only have to raise their deposit rates but also face deposit outflows, lowering their profitability and potentially decreasing bank intermediation capacity.

To fully explore this trade-off, in Section 3 we extend the static framework and embed it into a dynamic general equilibrium model with a set of features that are particularly relevant in this context: a banking sector that intermediates between households and firms, financial frictions that imply that bank profitability influences credit supply, a corporate production sector, a bond market that can substitute for bank financing, and nominal price rigidities building on the New Keynesian tradition. In Section 4, we tightly calibrate the model to U.S. empirical evidence and successfully match loan and bond spreads, as well as historical deposit rates for various levels of the policy rate. Replicating the empirical dependence of the deposit spread on the policy rate is a feature that is unique to our model and that is particularly important for our main results.

In Section 5, we use the model as a laboratory to explore the effects of introducing a CBDC and its role in monetary policy transmission. We first investigate how the impact of CBDC introduction varies with the level of CBDC remuneration. Interestingly, the welfare change displays an inverted U-shape. If a CBDC pays a very low interest rate, households hold a negligible amount of CBDC in their portfolios and banks' deposit market power is

largely unaffected, limiting the potential gains from CBDC introduction. By contrast, if a CBDC pays a very high interest rate, households flock to the CBDC, deposits pour out of the banking sector, and bank equity and bank lending contract substantially. As a result, the welfare impact of a CBDC turns negative as the bank disintermediation effect that leads to lower aggregate investment and output dominates the CBDC's beneficial effects. Thus, the model delivers a unique optimal CBDC rate. For our baseline economy that is calibrated to U.S. data, this rate is different from zero and lies at around 0.8% per year.

Next, instead of studying the introduction of a CBDC for a specific economy, we analyze the effects of introducing CBDCs in many economies that differ solely in the level of their steady-state policy rate. To start, we assess the introduction of a CBDC that pays zero interest. For a large range of negative and positive policy rates, we find positive welfare gains; those gains are smaller in high interest rate economies where households would hold only small amounts of CBDC and consequently bank deposit market power would barely be impacted. While encouraging, the previous exercise conceals the fact that a remunerated CBDC could lead to substantially higher welfare gains.

To explore this possibility, we determine the CBDC rate that maximizes welfare for each of these economies. For policy rates below 1%, the optimal CBDC rate is slightly negative. For policy rates above 1%, it lies between 80 and 120 basis points below the policy rate. We show that this welfare-maximizing CBDC rate can be well approximated by a simple rule of thumb that takes the maximum between 0% and the steady-state policy rate minus 1%. The simplicity of this rule is appealing, as it can be applied to many economies that differ substantially in their level of interest rates and it avoids the political-economy concerns that could arise when a CBDC pays negative interest. Thus, our results show that introducing a CBDC with remuneration is beneficial, especially for countries with high levels of interest rates. This finding resonates with recent surveys that show that around half of the central banks considering a CBDC remain open to the possibility of non-zero remuneration (see, for example, Figure 7a of the latest BIS survey on CBDC, Illes et al., 2025).

Introducing a CBDC with such a remuneration schedule has far-reaching effects on the banking system in our model. Particularly striking is how banks' deposit market power is curtailed in high interest rate environments. In fact, for the range of policy rates between 2% and 7%, we find that the positive relation between the deposit spread and the level of the policy rate vanishes after the introduction of a CBDC with the welfare-maximizing rate, and that the deposit spread stabilizes at around 70 basis points. This connects with the intuition from our static model: while cash is a weak competitor to deposits at high interest rates, a remunerated CBDC can substantially curtail bank deposit market power.

The increased competitiveness in deposit markets at high interest rates is also reflected in the welfare changes from CBDC introduction across policy rates. For policy rates below 2%, we find positive but modest welfare gains of around 0.25%—measured as the multiplicative consumption-equivalent variation required to keep the representative household indifferent between the pre-CBDC and the post-CBDC steady states. However, this number increases in high interest rate environments. For example, for a policy rate of 6%, we find a sizable welfare gain of around 1%. To put this number into context, it is equivalent to an additional $1200 per year for each household, given U.S. consumption expenditures in 2025.

While our model does not explicitly allow for certain changes in the structure of the banking sector such as bank entry or exit, it is still able to account for such adjustment patterns through endogenous changes in the size of the banking system. This mechanism works as follows. After a CBDC is introduced, banks initially experience lower profits as their deposit spreads decline. In turn, this pushes down bank equity and intermediation capacity, moving the banking sector up a downward sloping aggregate loan demand curve at higher loan rates and reduced quantities, and, as a result, the banking sector shrinks in size. Banks' return on equity stabilizes at its pre-CBDC level, as the decrease in deposit profitability is offset by an increase in loan profitability. This is a realistic feature of the model, as bank returns should not diverge from economy-wide returns in the long run.

Finally, we explore the role of CBDCs in potentially altering the response of an economy to typical business cycle innovations. Across a wide range of CBDC remuneration schedules, the reactions of various macroeconomic indicators to standard monetary policy and technology shocks are remarkably similar. Thus, even though the introduction of a CBDC can lead to significant welfare effects and changes in the financial landscape, responses to transitory shocks remain roughly unaltered. This is due to the fact that the main neoclassical channels of interest rate transmission in our model—the intertemporal substitution channel and the investment-based channels—are not significantly altered by the presence of CBDC or its remuneration schedule.

Having established our main results, Section 6 discusses additional aspects related to CBDC implementation. First, we decompose the welfare impacts of CBDC into the three channels discussed above: liquidity benefits, curtailing commercial bank monopoly power in deposits, and the bank disintermediation effect, finding that they are of roughly similar importance for our results. Second, we perform an alternative calibration for the Euro Area and find that the gains from introducing a CBDC are slightly lower than those for the United States due to the higher importance of banks in overall lending in the Euro

Area. Third, we take into account the costs of issuing and operating a CBDC system and show that our main results remain largely unchanged, since even cost figures above the upper range of existing estimates are still small relative to the welfare gains we obtain. Fourth, we discuss how bank reserves and wholesale or interbank funding would be affected by the introduction of a CBDC in an extended framework where banks had access to such alternative funding sources. Fifth, we discuss bank taxes/subsidies as an alternative to curtailing bank monopoly power and how they differ from introducing a CBDC. Finally, we briefly touch upon CBDC holding limits.

Related Literature. Our paper contributes to the new and rapidly emerging literature on the macroeconomic effects of CBDCs. Chapman et al. (2023) and Infante et al. (2023) survey the recent literature. In particular, our work is closely related to studies that examine the impact of CBDCs on bank disintermediation in a general equilibrium macroeconomic framework.

Most existing studies base their analysis on the New Monetarist approach. For example, Keister and Sanches (2022) show that a CBDC causes bank disintermediation as it crowds out bank deposits, leading to a decline in investment. However, they find that CBDC introduction often raises welfare by improving payment efficiency. Williamson (2022) finds that a CBDC can be welfare-improving as it promotes more efficient safe asset usage and helps mitigate a capital over-accumulation problem. In contrast to these models with competitive banking, Andolfatto (2021) considers a model with monopolistic banks and finds that the introduction of a CBDC can increase a bank's deposit rate and thus increase deposit financing while not necessarily impacting bank lending. Chiu et al. (2023) use a model of payments where banks engage in oligopolistic competition in the deposit market. In their model, the introduction of a CBDC *crowds in* bank deposits as long as the CBDC rate is not set too high. This effect is due to the assumption of perfect substitutability between deposits and CBDC. Relative to these contributions, we consider a New Keynesian dynamic stochastic general equilibrium (DSGE) model with imperfect substitutability between bank deposits and CBDC, bank market power in deposits and loans, and where bank profitability matters for bank lending.

Up to this point, relatively few papers have studied the macroeconomic effects of introducing CBDC in a DSGE model of the type that is commonly used by central banks. Most prominently, Burlon et al. (2023) find that the introduction of CBDC can lead to substantial welfare gains. In comparison, our model features bank market power in deposit markets, which gives rise to the endogenous deposit spread that we highlight, as well as nonbank lending through the bond market. As a result, our model allows for two

additional realistic channels through which CBDC could lead to potentially higher welfare gains. Barrdear and Kumhof (2022) find that CBDC issuance of 30% of GDP against government bonds could lower the real interest rate and thus increase GDP by 3%.

Other papers study optimal monetary policy and CBDC design. Brunnermeier and Niepelt (2019) formulate conditions under which a swap of private money for CBDC is irrelevant to economic allocations. Agur et al. (2022) consider the optimal design of CBDC in the presence of network effects. Niepelt (2023) studies the optimal quantity of CBDC in a growth and business cycle model where banks are monopsonists in deposit markets. He finds that the welfare-maximizing share of CBDC in payments generally exceeds that of deposits. In comparison, our framework features nominal rigidities, bank market power in loans, nonbank lending, and a role for bank profitability to determine credit supply.

The welfare gains of introducing CBDC may be higher if the bank disintermediation effect is dampened, which may occur for two reasons. Using a banking industry equilibrium model, Whited et al. (2023) show that banks largely replace lost deposits with wholesale funding, such that bank lending only contracts by a fourth of the deposits lost. Relatedly, Abad et al. (2025) find that banks mainly decrease their excess reserves when deposits leave, as opposed to contracting their lending. In our framework, banks are able to replace lost deposits with borrowing from the central bank or wholesale funding. However, unlike deposits, these alternative funding sources do not carry a spread that is favorable to banks. Therefore, the banking sector endogenously contracts and bank lending decreases. As our focus lies on analyzing aggregate welfare effects, we also emphasize that it is not necessarily bank lending disintermediation per se that matters but rather the change in overall lending. For example, if firms can easily substitute from bank to nonbank borrowing, bank disintermediation can be large but the change in total lending, and hence output, can be comparatively muted.

The modeling differences that we highlight distinguish our paper from the literature. They allow us to assess the trade-off between the key benefits of CBDC introduction (liquidity provision and increased deposit-market competition) and its detrimental aspects (the bank disintermediation effect) across levels of the policy rate and the CBDC rate, a dimension that has not been explored by any previous paper.[3]

[3]Central banks also mention other rationales for introducing CBDC that we do not cover in this paper. Financial inclusion, payments resilience, lowering remittance costs, or reducing reliance on foreign actors are aspects that may or may not be truly ameliorated by the introduction of CBDC. Quantifying these additional effects is an interesting path for future research but is beyond the scope of our closed-economy representative-agent model.

2 A Static Bank Deposit Model

How does the introduction of a CBDC affect the deposit rate and its spread relative to the policy rate? And how does this relationship change with the level of the policy rate and the interest rate on CBDC? In this section, we present a static partial equilibrium model of deposit intermediation with monopolistic banks to answer these questions. This simple model facilitates analytical tractability and helps to build intuition for the results of the larger quantitative general equilibrium model that we discuss in Section 3.

2.1 Deposit Supply Functions

To start, we take the household's deposit supply schedule as given. Section 3 shows how such a schedule can be formally derived from the household's optimization problem. The household has access to three liquidity-providing instruments: cash (m), aggregate deposits (d), and CBDC. Their returns are zero, i^d, and i^{cbdc}, respectively. The aggregate deposit supply function is

$$d = \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}} \right)^{\theta} \mathcal{L}, \tag{2.1}$$

where θ is the elasticity of substitution between the three aggregate liquidity-providing instruments, γ_d is described below, and $\mathcal{L}$ is the real aggregate liquidity supplied by the household, which we take as given for now and endogenize in Section 3. Equation (2.1) specifies that deposit supply depends positively on the ratio of the gross deposit rate to the gross rate on liquid instruments, defined as

$$1+i^{\mathcal{L}} = \left(\gamma_m + \gamma_d (1+i^d)^{\theta+1} + \gamma_{cbdc} (1+i^{cbdc})^{\theta+1} \right)^{\frac{1}{\theta+1}}. \tag{2.2}$$

The coefficients γ_m, γ_d, and γ_{cbdc} determine the importance of each of the instruments to the household due to exogenous non-interest-rate characteristics, and they satisfy $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. Aggregate deposits d, in turn, are comprised of deposits in n individual banks, each of which is indexed by j. Bank j pays a deposit rate of i_j^d and faces an individual deposit supply function given by

$$d_j = \frac{1}{n} \left(\frac{1+i_j^d}{1+i^d} \right)^{\varphi^d} d, \tag{2.3}$$

where φ^d is the elasticity of substitution between different banks. Equation (2.3) indicates that the supply of deposits to bank j depends positively on the ratio of its gross deposit rate to the aggregate gross deposit rate, which is defined as

$$1 + i^d = \left(\sum_{j=1}^{n} \frac{1}{n} (1 + i_j^d)^{\varphi^d + 1} \right)^{\frac{1}{\varphi^d + 1}}. \tag{2.4}$$

2.2 Banks

At the beginning of the period, each individual bank is endowed with equity f_j and issues deposits d_j. The bank uses these funds to finance its holding of reserves h_j, which pay the policy rate i. For simplicity, reserves are the only asset that banks invest in, an assumption that we relax in our full model. Bank j's balance sheet condition is therefore:

$$h_j = f_j + d_j. \tag{2.5}$$

The bank maximizes its end-of-period equity

$$\max_{i_j^d, d_j, h_j} \quad (1 + i) h_j - (1 + i_j^d) d_j,$$

subject to the deposit-supply equations (2.1)-(2.4) and the balance sheet constraint (2.5). Each bank has some monopoly power, and it chooses the interest rate it pays on deposits, the amount of deposits it takes on, and how many reserves to hold. The first-order condition for this bank problem is

$$1 + i_j^d = \frac{\epsilon_j^d}{\epsilon_j^d + 1} (1 + i), \tag{2.6}$$

where ϵ_j^d is the *endogenous* elasticity of deposits with respect to the deposit rate, that is, $\epsilon_j^d \equiv \partial \ln d_j / \partial \ln(1 + i_j^d)$; see Appendix A for derivations. Equation (2.6) highlights that bank j sets its deposit rate as a markdown on the policy rate. Assuming all banks are symmetric, we can express the endogenous elasticity of the representative bank as

$$\epsilon^d = \frac{n-1}{n} \varphi^d + \frac{\theta}{n} (1 - \omega_{\mathcal{L}}^d), \tag{2.7}$$

where

$$\omega_{\mathcal{L}}^{d} = \frac{(1+i^{d})d}{(1+i^{\mathcal{L}})\mathcal{L}} = \gamma_{d}\left(\frac{1+i^{d}}{1+i^{\mathcal{L}}}\right)^{\theta+1} \tag{2.8}$$

is the endogenous share of liquidity that stems from deposits at the end of the period, which we label the "endogenous deposit share" for short. When cash, deposits, and CBDC pay the same interest rate, the endogenous share happens to coincide with the exogenous share, $\omega_{\mathcal{L}}^{d} = \gamma_{d}$.

Equation (2.7) shows that the endogenous elasticity of bank deposits with respect to the deposit rate is a combination of two elasticities. With weight $(n-1)/n$, it simply reflects the exogenous elasticity φ^{d} with which depositors substitute across different banks. With the complementary weight $1/n$, it depends on how aggregate deposit supply reacts to changes in the aggregate deposit rate, a reaction which individual banks partially internalize due to their market power and non-infinitesimal size.[4] Given that all banks face the same endogenous elasticity, equation (2.6) can be expressed as

$$\frac{i-i^{d}}{1+i^{d}} = \frac{1}{\epsilon^{d}}, \tag{2.9}$$

where $(i-i^{d})/(1+i^{d})$ represents the spread that banks make when they accept deposits at rate i^{d} and keep them at the central bank earning the policy rate, normalized by $1+i^{d}$. This deposit spread is solely determined by the endogenous deposit elasticity. Taken together, equations (2.2) and (2.7)-(2.9) form a system that determines i^{d}, ϵ^{d}, $\omega_{\mathcal{L}}^{d}$, and $i^{\mathcal{L}}$.

2.3 How Central Bank Interest Rates Affect Deposit Rates

We first inspect how interest rates controlled by the central bank, namely the rate on reserves (i.e., the policy rate) and the CBDC interest rate, affect the deposit rate and the deposit spread.

Proposition 1.

1. *The deposit rate increases with the policy rate and the CBDC rate.*
2. *The deposit spread increases with the policy rate but decreases with the CBDC rate.*
3. *Aggregate deposits increase with the policy rate but decrease with the CBDC rate.*

Proof: see Appendix A.1.

[4] Atkeson and Burstein (2008) derive a similar equation, but their focus is on the goods market, whereas we study bank deposits.

The result on the deposit rate is intuitive, it shows the spillover from the central bank's policy instruments to the rates that are relevant for banks and households. But why do the two rates have opposite effects on the deposit spread and the amount of deposits? Equations (2.7), (2.8), and (2.9) are the key expressions that capture the transmission mechanism. When the rate on reserves increases, banks pass a fraction of this higher rate to their depositors. A higher deposit rate increases the endogenous deposit share $\omega^d_{\mathcal{L}}$, which has two effects. First, a higher deposit share translates directly into more aggregate deposits, given the exogeneity of aggregate liquidity. Second, a higher deposit share lowers the endogenous elasticity ϵ^d and increases the deposit spread. With a higher policy rate, banks gain market power relative to alternative liquid instruments and therefore charge a higher spread. This mechanism is also present in Drechsler et al. (2017). On the other hand, when the CBDC rate increases, CBDC poses more competition to banks and the endogenous share decreases, lowering both aggregate deposits and the deposit spread.

2.4 Effects of Introducing CBDC

Next, we turn to the core question of interest: what happens to the deposit rate and the deposit spread when the central bank introduces a CBDC? We capture the introduction of a CBDC by changing the CBDC interest rate from -100% to some higher percent that is roughly in the vicinity of 0%. We choose -100% as a starting point because that corresponds to the case where CBDC is not used at all in our larger DSGE model.

According to Proposition 1, the introduction of a CBDC increases the deposit rate, decreases the deposit spread, and induces an outflow of deposits. Based on a calibration that corresponds to the one used in Section 3, Panel A of Figure 2.1 plots the change in the deposit spread when CBDC is introduced with a 0% interest rate, as a function of the policy rate. Interestingly, it displays a U-shape. Per equations (2.7), (2.8), and (2.9), the underlying intuition works through the endogenous elasticity via the endogenous deposit share $\omega^d_{\mathcal{L}}$, which is plotted in Panel B of Figure 2.1. When CBDC pays a 0% interest rate and the policy rate is high, CBDC and cash barely compete with deposits, and hence $\omega^d_{\mathcal{L}}$ is close to one regardless of whether or not a CBDC exists. Therefore, the introduction of a CBDC leaves $\omega^d_{\mathcal{L}}$ mostly unaffected, and hence the deposit spread remains roughly unchanged. In the other extreme, for a fairly negative policy rate, deposits are undesirable compared with cash or CBDC. Therefore, $\omega^d_{\mathcal{L}}$ is close to zero regardless of the existence of CBDC. Deposits and CBDC are good substitutes for each other mostly when the policy rate is at intermediate levels. In this case, introducing a CBDC affects the endogenous deposit share and hence the endogenous elasticity of deposits substantially. Therefore,

Figure 2.1: Panel A: Change in the deposit spread following the introduction of a CBDC across different values of the policy rate. Panel B: Endogenous deposit share ($\omega^d_{\mathcal{L}}$) across different values of the policy rate before and after the introduction of CBDC. The figure uses the baseline calibration described in Section 4.

the deposit spread drops the most for moderate levels of the policy rate.

The U-shape is not unique to a CBDC that pays a zero interest rate. Panel A of Figure 2.2 shows that this shape holds as long as CBDC pays a constant interest rate. A higher interest rate on CBDC shifts the minimum of the curve towards the southeast: it increases the policy rate where CBDC introduction affects the deposit spread the most while also increasing the maximum change in the spread in absolute value.

Alternatively, when the CBDC rate is pegged to the policy rate, the U-shape disappears, as shown in Panel B of Figure 2.2. In this case, the change in the deposit spread is a decreasing function of the policy rate. This occurs because CBDC becomes more competitive with deposits the higher the policy rate is.

Thus, this simplified model points to an important trade-off of a potential CBDC introduction. While such a policy can benefit households and shield them from the monopolistic power of banks, it can lower deposit spreads and therefore affect commercial bank profitability negatively. In dynamic models where commercial bank profitability impacts bank equity and credit supply, a fall in bank profitability can negatively affect the economy. In the following section, we embed the static model into a general equilibrium model to quantify this trade-off and further study aggregate welfare effects.

Figure 2.2: Change in the deposit spread following the introduction of a CBDC, across different values of the policy rate, for different choices on the CBDC interest rate (i^{cbdc}). Panel A depicts a CBDC that pays a constant interest rate; Panel B depicts a CBDC that pays the policy rate with a fixed spread.

3 The DSGE Model

In this section, we introduce a full-fledged DSGE model for quantitative analyses. The key players in the model are a representative household, banks with monopoly power, a production sector, and a government.

The deposit side of the banking sector builds upon the ingredients laid out in Section 2. In addition, banks also issue corporate loans and face several operational costs. The household has access to four saving instruments: bonds, cash, bank deposits, and CBDC, where the last three instruments provide liquidity services with imperfect substitution.

The production sector consists of a representative intermediate good firm, a representative capital producer, monopolistically competitive retail firms, and a representative final good producer. The intermediate good firm purchases capital from the capital producer and combines it with labor from the household to produce an intermediate good. Its capital input is aggregated from two types of capital with non-unitary substitution: "non-pledgeable capital," which is financed by unsecured bond borrowing, and "pledgeable capital," which is financed through bank loans.

Retail firms face the standard Calvo price rigidity and transform the intermediate good into differentiated retail goods, which are then aggregated into a final good by the

final good producer. The government includes a central bank that conducts monetary policy and a fiscal authority with a balanced budget.

3.1 Household

Setup. The household's lifetime utility is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(C_t) - v(N_t) \right),$$

where β is the discount factor, C_t is consumption, N_t is labor supply, and u and v are functions with the standard properties. The household's budget constraint is given by

$$P_t C_t + B_t + \Phi(\mathcal{L}_t) P_t \quad = \quad W_t N_t + AH_{t-1} + T_t,$$

where P_t is the aggregate price level, B_t are nominal bond holdings, W_t is the nominal wage, and $\Phi(\mathcal{L}_t)$ is described below. T_t captures transfers that are exogenous from the household's perspective, including net government transfers and profits from firms and banks. AH_{t-1} refers to "assets in hand" that the household enters period t with, given by

$$AH_{t-1} = M_{t-1} + (1 + i_{t-1}) B_{t-1} + \sum_{j=1}^{n} (1 + i^d_{j,t-1}) D_{j,t-1} + (1 + i^{cbdc}_{t-1}) CBDC_{t-1}.$$

The household can save in cash (M_t), bonds (B_t), deposits with any of the n different commercial banks ($D_{j,t}$), and CBDC ($CBDC_t$) if available, where capital letters denote nominal terms. The associated net nominal returns for these instruments are zero, i_t, $i^d_{j,t}$, and i^{cbdc}_t, respectively. The variable $\mathcal{L}_t$ in the budget constraint aggregates the various liquidity-providing instruments (cash, deposits, and CBDC), and is defined as

$$\mathcal{L}_t \quad = \quad \left(\gamma_m^{-\frac{1}{\theta}} m_t^{\frac{\theta+1}{\theta}} + \gamma_d^{-\frac{1}{\theta}} d_t^{\frac{\theta+1}{\theta}} + \gamma_{cbdc}^{-\frac{1}{\theta}} cbdc_t^{\frac{\theta+1}{\theta}} \right)^{\frac{\theta}{\theta+1}}, \tag{3.1}$$

where lowercase letters denote real variables (e.g., $m_t = M_t / P_t$). The parameter θ is the elasticity of substitution between liquid instruments and $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. Additionally, real deposits d_t are an aggregate of deposits in n banks:

$$d_t = \left(\sum_{j=1}^{n} \alpha_j^{-\frac{1}{\varphi^d}} d_{j,t}^{\frac{\varphi^d+1}{\varphi^d}} \right)^{\frac{\varphi^d}{\varphi^d+1}}, \tag{3.2}$$

where $\sum_{j=1}^{n} \alpha_j = 1$ and $\varphi^d \geq \theta$. The fact that cash, deposits, and CBDC are not perfect substitutes within $\mathcal{L}_t$ captures the possibility that the household uses them for different types of transactions because of their different properties. For example, bank deposits and CBDC are useful for online transactions, while cash is not; cash provides better anonymity than deposits and CBDC; cash and CBDC are government-backed while bank deposits are not necessarily insured; cash is more likely to be subject to theft. For these reasons, among others, the representative household might want to hold a combination of liquidity-providing instruments instead of simply holding the one with the highest return. A similar argument holds for deposits from different banks.[5]

Lastly, $\Phi(\mathcal{L}_t)$ captures a potentially nonlinear cost function of acquiring liquidity. If $\Phi(\cdot)$ is linear and $\theta \to \infty$ (making the three liquid instruments perfect substitutes), then $\Phi(\mathcal{L})P = M + D + CBDC$, which is the standard way that savings instruments without liquidity benefits enter the budget constraint. By contrast, the $\Phi(\cdot)$ function that we use in our baseline specification is flexible enough to capture the feature that when the household has few liquid instruments the cost of acquiring liquidity could be less than one-for-one, $\Phi(\mathcal{L}_t) < \mathcal{L}_t$, reflecting the convenience benefit of holding liquidity. Eventually, if agents get "satiated" with liquidity services, it can instead be the case that $\Phi(\mathcal{L}_t) > \mathcal{L}_t$.[6] We choose to introduce $\Phi(\cdot)$ directly in the budget constraint for simplicity. However, as shown in Appendix B.2, one can obtain the same first-order conditions for the liquidity-providing instruments by allowing them to enter the utility function instead.[7] Additionally, the CES-like expressions that we have used in equations (3.1) and (3.2) can be microfounded as in Ulate (2021), which would add further complexity to the model without providing additional insights.

Equilibrium Conditions. Our setup delivers convenient equilibrium conditions. First, the optimality conditions with respect to labor and bonds are the usual intratemporal condition for labor supply and the Euler equation:

$$v'(N_t) = u'(C_t)\left(\frac{W_t}{P_t}\right), \tag{3.3}$$

[5]Note that, unlike the traditional CES aggregator, the exponents within the $\mathcal{L}_t$ and d_t aggregators are greater than one instead of smaller than one. This occurs because these aggregators enter the budget constraint instead of the utility function. Therefore, they must be convex (instead of concave) to prevent the household from bunching its choice into a single liquidity-providing instrument or a single bank.

[6]Balloch and Koby (2019) use a related cost function of liquidity in the context of negative nominal interest rates in Japan. The "satiation" embedded in our baseline functional form for $\Phi(\mathcal{L}_t)$ is similar to the one described in Rognlie (2016), but for liquidity as a whole rather than just for cash.

[7]This holds as long as one assumes a non-separable utility function between consumption and liquidity in the style of Greenwood et al. (1988), as shown in Appendix B.2.

$$\frac{u'(C_t)}{P_t} = \beta(1+i_t)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right). \tag{3.4}$$

Next, the holding schedules of the liquidity-providing instruments are

$$m_t = \gamma_m\left(\frac{1}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t, \tag{3.5}$$

$$d_t = \gamma_d\left(\frac{1+i_t^{d}}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t, \tag{3.6}$$

$$cbdc_t = \gamma_{cbdc}\left(\frac{1+i_t^{cbdc}}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t. \tag{3.7}$$

These holding schedules are well defined even for negative values of any of the interest rates. The interest rate for liquidity and aggregate deposits are defined as

$$1+i_t^{\mathcal{L}} \equiv \left(\gamma_m + \gamma_d(1+i_t^d)^{\theta+1} + \gamma_{cbdc}(1+i_t^{cbdc})^{\theta+1}\right)^{\frac{1}{\theta+1}} \tag{3.8}$$

$$1+i_t^{d} \equiv \left(\sum_{j=1}^{n}\alpha_j(1+i_{j,t}^d)^{\varphi^d+1}\right)^{\frac{1}{\varphi^d+1}}.$$

Furthermore, the amount of deposits supplied by the household to an individual bank is

$$d_{j,t} = \alpha_j\left(\frac{1+i_{j,t}^d}{1+i_t^d}\right)^{\varphi^d} d_t, \tag{3.9}$$

and the equilibrium condition for the aggregator $\mathcal{L}_t$ is as follows:

$$\frac{1+i_t^{\mathcal{L}}}{1+i_t} = \Phi'(\mathcal{L}_t). \tag{3.10}$$

Appendix B.1 provides details on the equilibrium-conditions derivation. Equations (2.1)-(2.4) are a special case of the equilibrium conditions above. Besides being static and considering $\mathcal{L}$ exogenous, Section 2 imposes the symmetry restriction that $\alpha_j = 1/n\ \forall j$.

3.2 Intermediate Good Firm

The intermediate good firm uses labor and capital to produce intermediate output:

$$Y_t^m = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{3.11}$$

where $0 < \alpha < 1$, Y_t^m is the amount of intermediate output produced, A_t is productivity, and K_t is capital input. The firm purchases capital from a capital producer and finances its purchases via two possible channels. It borrows from the bond market to finance capital that cannot be used as collateral, denoted non-pledgeable capital K_t^{NP}, reflecting the empirical observation that bond borrowing is typically unsecured (Schwert, 2020). Alternatively, the firm can borrow from banks to purchase pledgeable capital K_t^P that can be used as collateral. Aggregate capital is a CES combination of these two types:

$$K_t = \left((1-\psi)^{\frac{1}{\theta^k}} (K_t^{NP})^{\frac{\theta^k-1}{\theta^k}} + \psi^{\frac{1}{\theta^k}} (K_t^P)^{\frac{\theta^k-1}{\theta^k}} \right)^{\frac{\theta^k}{\theta^k-1}}, \tag{3.12}$$

where θ^k captures the elasticity of substitution between the two types. K_t^P is itself an aggregate of the pledgeable capital financed by each of the n banks:

$$K_t^P = \left(\sum_{j=1}^{n} (\alpha_j^l)^{\frac{1}{\varphi^l}} (K_{j,t}^P)^{\frac{\varphi^l-1}{\varphi^l}} \right)^{\frac{\varphi^l}{\varphi^l-1}},$$

where φ^l is the loan elasticity of substitution among banks and α_j^l captures the exogenous importance of a particular bank in the loan portfolio, with $\sum_{j=1}^{n} \alpha_j^l = 1$ and $\varphi^l \geq \theta^k$.

Capital is predetermined. In period $t-1$, the intermediate good firm borrows from the bond market or banks in order to purchase capital for next period's production at price Q_{t-1}. At time t, it sells its depreciated capital stock $(1-\delta)$ back to the capital producer after production. Meanwhile, it pays back the lenders who charge different interest rates: bank j charges the loan rate $i_{j,t-1}^l$, while the bond market charges the risk-free rate i_{t-1} plus a spread ϱ. The intermediate good firm's period t profit is

$$\begin{aligned} \Pi_t^m &= P_t^m Y_t^m - W_t N_t + (1-\delta) Q_t \sum_{j=1}^{n} K_{j,t}^P + (1-\delta) Q_t K_t^{NP} \\ &- \sum_{j=1}^{n} (1 + i_{j,t-1}^l) Q_{t-1} K_{j,t}^P - (1 + i_{t-1} + \varrho) Q_{t-1} K_t^{NP}. \end{aligned}$$

The intermediate good firm maximizes the present value of profits (discounted using the household's stochastic discount factor) by choosing labor and capital inputs. The associated optimality conditions are given in Appendix B.3, and they depend on the real wage, as well as on the effective one-period user costs of aggregate capital, pledgeable capital, and non-pledgeable capital, which we denote with z_t, z_t^P, and z_t^{NP}, respectively.

3.3 Capital Good Producer

The capital producer faces the capital accumulation equation:

$$\left[K_{t+1}^{NP} + \sum_{j=1}^{n} K_{j,t+1}^{P}\right] = (1-\delta)\left[K_{t}^{NP} + \sum_{j=1}^{n} K_{j,t}^{P}\right] + I_t\left(1 - \Xi\left(\frac{I_t}{I_{t-1}}\right)\right), \tag{3.13}$$

where I_t is overall investment and the function $\Xi(\cdot)$ captures investment adjustment costs and satisfies $\Xi(1) = \Xi'(1) = 0$ and $\Xi''(1) \geq 0$. The problem of the capital producer in t is:

$$\max_{I_t} \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau}\left[Q_{t+\tau} I_{t+\tau}\left(1 - \Xi\left(\frac{I_{t+\tau}}{I_{t+\tau-1}}\right)\right) - P_{t+\tau} I_{t+\tau}\right],$$

where $\Lambda_{t,t+\tau}$ denotes the household's stochastic discount factor for discounting nominal flows from $t+\tau$ back to t. This problem's first-order condition is given in Appendix B.4.

3.4 Banks

The liability side of the bank balance sheet is similar to the one in Section 2.2; on the asset side, we also consider the possibility of lending to the production sector. Therefore, the nominal balance sheet constraint of bank j now takes the following form

$$L_{j,t} + H_{j,t} = F_{j,t} + D_{j,t}, \tag{3.14}$$

where $L_{j,t}$ represents lending to the intermediate firm, $H_{j,t}$ are reserves issued by the central bank, $F_{j,t}$ is bank equity, and $D_{j,t}$ are household deposits, all in nominal terms.[8]

Besides adding bank lending, we introduce three additional features. First, in each period, a bank returns an exogenous fraction, $1-\omega$, of its profits to the household as dividends and spends a fraction ς of its nominal net worth to operate the managerial side of the bank. This setup implies that bank equity is partially determined by bank profitability. Second, a bank pays a quadratic cost, denoted by $\Psi(L_{j,t}/F_{j,t})$, when its loan-to-equity ratio, $L_{j,t}/F_{j,t}$, deviates from a target value. This cost captures the idea that regulators discourage banks from having high levels of leverage by imposing punishments when banks breach certain capital requirements, while market forces incentivize banks to avoid levels of leverage that are too low. Together, the previous two assumptions imply that a fall in bank profitability stemming from the introduction of a CBDC can impact bank

[8] $H_{j,t}$ is allowed to be negative, representing borrowing from the central bank at the policy rate. See Appendix B.5.2 for the relevance of this assumption and Section 6.4 for further discussion.

equity, which in turn can affect bank lending. Finally, banks face exogenous costs of issuing loans, μ^l, and obtaining deposits, μ^d, expressed per dollar of loan or deposit issued. These costs are used to match the deposit and lending spreads without having to necessarily assume their existence is solely due to the presence of monopoly power.

With the assumptions described in the previous paragraph, the nominal resources that bank j has available when entering period $t+1$ are given by

$$S_{j,t+1} = (1 + i^l_{j,t} - \mu^l)L_{j,t} + (1 + i_t)H_{j,t} - (1 + i^d_{j,t} + \mu^d)D_{j,t} - \varsigma F_{j,t} - \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t}.$$

These resources have to be used either to pay dividends or as next-period equity: $S_{j,t+1} = F_{j,t+1} + DIV_{j,t+1}$, where dividends $DIV_{j,t+1}$ are a fraction $1-\omega$ of a bank's profit $X_{j,t+1}$:

$$DIV_{j,t+1} = (1-\omega)X_{j,t+1},$$

and profits $X_{j,t+1}$ are, in turn, defined as

$$X_{j,t+1} \equiv i_t F_{j,t} + (i^l_{j,t} - \mu^l - i_t)L_{j,t} + (i_t - \mu^d - i^d_{j,t})D_{j,t} - \Psi\left(\tfrac{L_{j,t}}{F_{j,t}}\right)F_{j,t} - F_{j,t}(1-\varsigma)\pi_{t+1}.$$

We define $X_{j,t+1}$ as the net profit before paying managerial costs but after adjusting for inflation $\pi_{t+1} \equiv P_{t+1}/P_t - 1$. The inflation adjustment is purely for convenience, because it delivers a clean and interpretable expression for the law of motion of real bank equity, which takes the form

$$\frac{F_{j,t+1}}{P_{t+1}} = \frac{F_{j,t}}{P_t}(1-\varsigma) + \omega\frac{X_{j,t+1}}{P_{t+1}}. \tag{3.15}$$

If $\omega = \varsigma = 0$, then a bank's real equity is constant. The larger ω is, the more bank equity depends on profits and the more volatile it becomes. Appendix B.5.1 shows that in any steady state of our model, pre- or post-CBDC, bank return on equity is the same, an important and useful feature of our model that we will return to in Section 5.1.

A bank seeks to maximize the present discounted value of future dividends that it returns to the household. Hence, bank j's problem is:

$$\max \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau+1} DIV_{j,t+\tau+1}.$$

As shown in Appendix B.5.2, the solution to the bank's problem can be broken down into a deposit and a loan sub-problem that we discuss next.

Deposit Sub-problem. The deposit sub-problem amounts to

$$\max_{i^d_{j,t}} (i_t - i^d_{j,t} - \mu^d) D_{j,t},$$

subject to the deposit supply schedule $D_{j,t}(i^d_{j,t})$ of the household. Assuming that a bank takes the decisions of all other banks as given, it sets its deposit rate as follows:

$$1 + i^d_{j,t} = \frac{\epsilon^d_{j,t}}{\epsilon^d_{j,t} + 1}(1 + i_t - \mu^d). \tag{3.16}$$

Expression (3.16) shows that banks set their gross deposit rate as a markdown on the gross policy rate minus the cost of issuing deposits. The markdown is determined by $\epsilon^d_{j,t}$, the endogenous elasticity of bank j's deposits with respect to its deposit rate:

$$\epsilon^d_{j,t} \equiv \frac{\partial d_{j,t}}{\partial(1 + i^d_{j,t})} \frac{1 + i^d_{j,t}}{d_{j,t}}.$$

For the case with identical banks, this endogenous elasticity is (see Appendix B.5.3):

$$\epsilon^d_t = \frac{n-1}{n}\varphi^d + \frac{1}{n}\left[(1 - \omega^d_{\mathcal{L},t})\theta + \omega^d_{\mathcal{L},t}\frac{\partial \ln \mathcal{L}_t}{\partial \ln(1 + i^{\mathcal{L}}_t)}\right], \tag{3.17}$$

where $\omega^d_{\mathcal{L},t}$ is again the endogenous deposit share

$$\omega^d_{\mathcal{L},t} \equiv \frac{(1 + i^d_t)d_t}{(1 + i^{\mathcal{L}}_t)\mathcal{L}_t} = \gamma_d \left(\frac{1 + i^d_t}{1 + i^{\mathcal{L}}_t}\right)^{\theta+1}. \tag{3.18}$$

Note that we can recover the expression in equation (2.7) from equation (3.17) when $\partial \ln \mathcal{L}_t / \partial \ln(1 + i^{\mathcal{L}}_t) = 0$, which is imposed in Section 2, where $\mathcal{L}$ is assumed to be constant. Thus, even in the larger model, the interpretation of ϵ^d_t remains similar: it is a weighted average of the exogenous elasticities φ^d and θ, as well as $\partial \ln \mathcal{L}_t / \partial \ln(1 + i^{\mathcal{L}}_t)$, where the weights for the last two terms are endogenous and vary with the introduction of CBDC.

Loan Sub-problem. The loan sub-problem of bank j is:

$$\max_{i^l_{j,t}} (i^l_{j,t} - i_t - \mu^l) L_{j,t} - \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right) F_{j,t},$$

subject to the loan demand schedule of the intermediate firm and $L_{j,t} = Q_t K^P_{j,t+1}$. As opposed to the *markdown* on the deposit rate, each individual bank sets its gross loan rate as a *markup* on the cost-adjusted policy rate:

$$1 + i^l_{j,t} = \frac{\epsilon^l_{j,t}}{\epsilon^l_{j,t} - 1}\left[1 + i_t + \mu^l + \Psi'\left(\frac{L_{j,t}}{F_{j,t}}\right)\right], \tag{3.19}$$

where $\epsilon^l_{j,t}$ denotes the endogenous loan elasticity of $l_{j,t}$ with respect to $1 + i^l_{j,t}$:

$$\epsilon^l_{j,t} \equiv -\frac{\partial l_{j,t}}{\partial(1 + i^l_{j,t})}\frac{1 + i^l_{j,t}}{l_{j,t}}.$$

As shown in Appendix B.5.4, for the case of identical banks, this elasticity is:

$$\epsilon^l_t = \left\{\frac{n-1}{n}\varphi^l + \frac{1}{n}\left[\theta^k(1 - \omega^{K_P}_{K,t}) + \frac{\omega^{K_P}_{K,t}}{1-\alpha}\right]\right\}\frac{Q_t}{P_t}\frac{1 + i^l_t}{1 + i_t}\frac{1}{z^P_t}, \tag{3.20}$$

where $\omega^{K_P}_{K,t}$ is the expenditure on pledgeable capital as a share of total capital expenditure

$$\omega^{K_P}_{K,t} \equiv \frac{z^P_t K^P_t}{z_t K_t} = \psi\left(\frac{z^P_t}{z_t}\right)^{1-\theta^k}. \tag{3.21}$$

Notice the parallels between equations (3.17) and (3.20). For both, the endogenous elasticity puts a weight $(n-1)/n$ on the exogenous elasticity (φ^d or φ^l). The remaining weight of $1/n$ is split between the two elasticities inside the square brackets: an elasticity of substitution (θ for deposits or θ^k for loans) and the elasticity of total liquidity ($\partial \ln \mathcal{L}_t / \partial \ln(1 + i^{\mathcal{L}}_t)$) or capital ($\partial \ln K / \partial \ln z = 1/(1-\alpha)$) with respect to its price.

3.5 Retail Firms and Final Good Producer

The setup of retail firms and the final good producer follows the typical modeling approach in the New Keynesian literature. A continuum of retail firms indexed by $s \in [0, 1]$ transform intermediate output Y^m_t into differentiated retail goods $Y_t(s)$, which are aggregated into a final good Y_t by the final good producer via a standard CES aggregator with an elasticity of substitution ϕ between the differentiated retail goods. The optimization problem of the final good producer implies the following demand function for good s and the aggregate price index:

$$Y_t(s) = \left(\frac{P_t(s)}{P_t}\right)^{-\phi} Y_t, \qquad P_t = \left(\int_0^1 P_t(s)^{1-\phi} ds\right)^{\frac{1}{1-\phi}}.$$

Each period, a retail firm is able to freely adjust its price with probability $1-\gamma$ as in the Calvo setup, and it chooses the optimal reset price P_t^* to solve:

$$\max_{P_t^*} \ \mathbb{E}_t \sum_{\tau=0}^{\infty} \gamma^\tau \beta^\tau \frac{u'(C_{t+\tau})}{u'(C_t)} \frac{P_t}{P_{t+\tau}} \left[P_t^* - P_{t+\tau}^m\right] Y_{t+\tau|t},$$

where $Y_{t+\tau|t}$ is the amount sold in period $t+\tau$ by a firm that last reset its price in period t. The optimality conditions for retail firms are given in Appendix B.6.

3.6 Government

Monetary policy is characterized by a Taylor rule with interest rate smoothing:

$$i_t = (1-\rho_i)\left(\bar{\imath} + \psi_\pi(\pi_t - \overline{\pi})\right) + \rho_i i_{t-1} + \epsilon_t^i, \tag{3.22}$$

where $\bar{\imath}$ is the steady-state nominal rate, $\rho_i \in [0,1]$ reflects interest rate inertia, and ϵ_t^i is an exogenous shock to monetary policy. Note that the policy rate is also the rate on reserves, which is the same as the return on bonds. For simplicity, we assume government spending is a constant fraction of output

$$G_t \ = \ gY_t. \tag{3.23}$$

We also assume that the government balances its budget period-by-period. Therefore, the lump sum transfers from the government to the household are given by the proceeds from seigniorage (covering cash, reserves, and CBDC) net of government expenditures. The balance sheet of the central bank, which endogenously incorporates any potential substitution across different central bank liabilities after CBDC introduction, is only necessary to derive the aggregate resource constraint, which is given in the following section.

3.7 Resource Constraint and Shocks

Output is divided between consumption, investment, government expenditure, and adjustment costs. The economy-wide resource constraint is thus given by

$$Y_t \ = \ C_t + I_t + G_t + \Gamma_t, \tag{3.24}$$

where Γ_t represents all additional costs:

$$\begin{aligned}\Gamma_t &= \mu^l \frac{L_{t-1}}{P_t} + \mu^d \frac{D_{t-1}}{P_t} + \varsigma \frac{F_{t-1}}{P_t} + \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)\frac{F_{t-1}}{P_t} + \varrho \frac{Q_{t-1}}{P_t} K_t^{NP} \\ &+ \Phi(\mathcal{L}_t) - \frac{M_t + D_t + CBDC_t}{P_t} + \Omega\left(\frac{CBDC_t}{P_t Y_t}\right). \end{aligned} \tag{3.25}$$

In the previous expression, Ω represents the per-period costs for the government of operating the CBDC, expressed as a function of the ratio of real CBDC to output. In our baseline specification we set $\Omega = 0$, but in Section 6.3 we discuss how realistic values for the costs of issuing or operating a CBDC do not significantly alter our conclusions. Finally, we assume that the technology process follows an AR(1):

$$A_t = A_{t-1}^{\rho_a} \exp(\epsilon_t^a). \tag{3.26}$$

The full set of dynamic equilibrium equations is given in Appendix B.7.

4 Calibration

We calibrate the model to the U.S. economy at a quarterly frequency. The parameters associated with the financial block are particularly important for the quantitative realism of the model. We lay out our calibration in four parts. First, we discuss parameters that are set externally. Next, we collect parameters related to the deposit side of the model, followed by the ones associated with the loan side, and finally we discuss all other bank parameters. Table 4.1 lists the full set of parameters, their values, and calibration targets.

4.1 Nonbank Parameters

The quarterly discount factor, β, is set to 0.995, giving an annualized policy rate of 2%, which is consistent with the low interest rates that prevailed in the United States before the COVID-19 pandemic. We use the standard functional forms for $u(c)$ and $v(n)$:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \qquad v(n) = \chi \frac{n^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}, \tag{4.1}$$

and set the intertemporal elasticity of substitution, $1/\sigma$, and the Frisch elasticity, η, both to one. The former is consistent with balanced growth in our model, while the latter is consistent with the upper bound for macro elasticities in Chetty et al. (2011). The disutility

from labor, χ, is chosen such that steady-state labor is normalized to one-third.

The capital income share, α, is one-third and the depreciation rate, δ, is 0.02 quarterly, or 8% annually. The functional form for the investment adjustment cost function is $\Xi(x) = \kappa_I/2 \cdot (x-1)^2$, where κ_I is set to 2 as in Sims and Wu (2021). We set the elasticity of substitution between differentiated retail goods, ϕ, to 6, which is consistent with a steady-state markup of 20%. The Calvo parameter, γ, capturing the probability that a retail firm is not allowed to adjust its price, is set to the typical value of 0.75, implying an average duration between price updates of one year. The Taylor rule parameters are set to standard values: the persistence parameter, ρ_i, is 0.8 and the response to inflation, ψ_π, is 1.5. Steady state inflation is assumed to be zero for simplicity. Finally, the ratio of government spending to GDP, g, is set to 0.2, roughly consistent with historical U.S. data.

4.2 Deposit Parameters

On the deposit side, it is important that our model matches empirical estimates for deposit rates at different levels of the policy rate, and therefore deposit spreads, to align the model with the strength of bank deposit market power in the data. Specifically, we target four moments: (i) a deposit rate of 0% at a policy rate of 0.5% (taken from Ulate, 2021), (ii) a deposit rate of 0.75% at a policy rate of 2%, (iii) a deposit rate of 1.25% at a policy rate of 3%, and (iv) a deposit rate of 2% at a policy rate of 4.5%, where the last three targets are estimated from historical Ratewatch data.[9] We match these four moments by jointly calibrating n, the number of banks, θ, the elasticity of substitution between different liquidity-providing instruments, φ^d, the elasticity of substitution between banks in deposits, and μ^d, the cost of issuing deposits. This exercise yields estimates of $n = 1.16$, $\theta = 554$, $\varphi^d = 661$, and $\mu^d = -0.0020$ (20 basis points quarterly).

Three points are noteworthy about these estimates. First, our calibration requires a fairly low value of n. While this estimate is not an integer, and it is certainly lower than the actual number of U.S. banks, we do not intend it to be taken literally.[10] Rather, it allows the model to match the relationship between the deposit rate and the policy rate (which is tightly related to bank market power) while remaining parsimonious and work-

[9] We compute a historic deposit rate series that is the empirical counterpart of the one in our model by using data from Ratewatch on checking and saving deposit rates and weighting those by the historical shares of such deposits based on data from the H.6 releases from the Federal Reserve Board of Governors (sample: 2000:M1-2020:M4). Comparing the resulting series to the federal funds rate yields approximately the calibration targets between the policy rate and the deposit rate stated in the text.

[10] Notice that once our model has been aggregated to n symmetric banks, there is no longer any constraint that n needs to be an integer.

Table 4.1: Calibration.

Param.	Value	Description	Target or source
Panel A. Nonbank			
β	0.9950	Discount factor	2% annual policy rate
χ	8.8487	Disutility of labor	One-third S.S. labor
η	1.0000	Frisch elasticity	Chetty et al. (2011)
σ	1.0000	Inverse of the I.E.S.	Balanced Growth
α	0.3333	Capital share	Standard
δ	0.0200	Depreciation rate	8% annual dep.
κ_I	2.0000	Investment adjustment cost	Sims and Wu (2021)
ϕ	6.0000	Elasticity of subs. b/t diff. goods	20% mark-up
γ	0.7500	Prob. of keeping prices fixed	One-year duration
ψ_π	1.5000	Inflation coefficient, Taylor rule	Standard
ρ_i	0.8000	Smoothing parameter, Taylor rule	Standard
g	0.2000	Steady state G/Y	Standard
Panel B. Deposit side			
n	1.1685	Number of banks	Deposit rate target #1
θ	554.21	E.o.S. between liquid instruments	Deposit rate target #2
φ^d	661.36	E.o.S. between banks in deposits	Deposit rate target #3
μ^d	-0.20%	Cost of issuing deposits	Deposit rate target #4
γ_m	0.3005	Importance of cash in liquidity	$\gamma_m + \gamma_d + \gamma_{cbdc} = 1$
γ_d	0.3990	Importance of deposits in liquidity	$D/\mathcal{L} = 0.8$ at $i = 2\%$
γ_{cbdc}	0.3005	Importance of CBDC in liquidity	$\gamma_{cbdc} = \gamma_m$ (Bidder et al.)
a	0.8764	Parameter in liquidity function Φ	$\mathcal{L}/Y = 2.4$ quarterly
b	1.0700	Parameter in liquidity function Φ	Estimation
q	-0.1615	Parameter in liquidity function Φ	S.S. relationship
Panel C. Loan side			
ψ	0.3000	Importance of pledgeable capital	Crouzet (2021)
ϱ	0.70%	Extra cost of corporate-bond borrowing	Schwert (2020)
μ^l	0.35%	Cost of issuing loans	Schwert (2020)
φ^l	40.013	E.o.S. between banks in loans	$i^l = i + \varrho \Rightarrow \theta^k = f(\varphi^l)$
θ^k	5.0000	Subs. between NP and P capital	Feasible region
Panel D. Joint bank side			
ω	0.6780	Fraction staying in bank	$L/F = \nu$ in S.S.
ς	0.0474	Bank managerial cost	2.25% S.S. quarterly ROE
ν	9.0000	Loan-to-equity ratio target	Ulate (2021)
κ	0.0012	Cost of deviating from target ratio	Ulate (2021)

Notes: This table contains the parameter values used in the calibration, together with their description and their source or target.

ing well within our larger general equilibrium model.[11] Second, the negative value of μ^d implies a "benefit" of issuing deposits instead of a cost. Reassuringly, the calibrated value is close to the one in Ulate (2021) of -0.0025. In reduced form, the negative μ^d could capture complementarities between deposit taking and lending, fees charged to depositors, or benefits of using a relatively stable source of funding (a similar feature is present in Abadi et al., 2023).[12] Third, while the values for the elasticities θ and φ^d may, at first glance, appear large, these elasticities vary with the model's time-frequency and are not comparable with elasticities computed with respect to net interest rates (which are more common in the literature). For example, our estimate for φ^d of 661 is equivalent to an annual elasticity in terms of net interest rates of approximately 2.[13]

The exogenous shares of cash, deposits, and CBDC (γ_m, γ_d, and γ_{cbdc}) are set to match two targets along with the model-implied restriction that $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. The first target is the pre-CBDC deposit-to-liquidity ratio $d/\mathcal{L}$ in steady state. We obtain an estimate for this ratio using historical data on checking deposits, savings deposits, and currency holdings, and constructing $\mathcal{L}$ based on equation (3.1) given our calibration for θ. For the sample 1975:Q1-2020:Q1, we find that $d/\mathcal{L}$ is approximately 0.8 on average. The second target is taken from surveys such as Bidder et al. (2024). When asked about their potential CBDC usage, people report that they would hold roughly the same amount of CBDC and cash if CBDC paid no interest, which implies $\gamma_{cbdc} = \gamma_m$. Appendix C.1 performs additional robustness exercises and discusses how our results change if we vary this target to alternative ratios like $\gamma_{cbdc} = 0.5\gamma_m$ or $\gamma_{cbdc} = 2\gamma_m$. As expected, the higher γ_{cbdc}, the greater the welfare benefits of introducing CBDC. Putting all of our targeted moments together, our baseline calibration yields $\gamma_m = \gamma_{cbdc} = 0.3005$ and $\gamma_d = 0.3990$.[14]

The cost-of-liquidity function is parameterized as $\Phi(\mathcal{L}) = a\mathcal{L}^b - q$. The elasticity

[11] In Appendix A.2, we obtain a closed-form expression for the pass-through of the policy rate to the deposit rate and show that the crucial parameter that governs this relation is the number of banks, n. Empirically, this pass-through is found to be less than unity. Drechsler et al. (2017), for example, document a pass-through of 0.39 among large banks and 0.46 on average (see pages 1821 and 1824 therein).

[12] Note that any further fixed costs of operating the deposit franchise are incorporated in the managerial costs of operating the bank, ς, which are substantial in our baseline calibration as described in the text.

[13] Since our model is at the quarterly frequency, we must divide our elasticity estimates by four to obtain annual estimates. Our annual estimate for φ^d would then be 165. Since this is an elasticity in gross rates, a bank that paid an annual deposit rate of 1.5% when the aggregate annual deposit rate is 1% would obtain $(1.015/1.01)^{165} \approx 2.25$ times the amount of deposits obtained by the "average" bank. To turn this into an elasticity in net interest rates, we solve for $(1.5/1)^{\zeta} = 2.25$, which yields $\zeta = 2$. This clarifies that our estimates are in the same ballpark as those in the literature. For example, Gerali et al. (2010) use $\zeta = 1.46$.

[14] An alternative to assess the demand for CBDC uses demand system approaches. For Canada, Li (2023) finds that people would hold between 4% and 20% of their liquid assets in CBDC when taking into account that banks adjust deposit rates. For the Euro area, Lambert et al. (2024) find similar estimates of 3% and 28%. Our model produces a number of 13% for a zero CBDC rate and a 2% policy rate (see Section 5.1), which reassuringly lies almost exactly in the middle of both of these independent estimates.

parameter b is calibrated starting from equilibrium condition (3.10),

$$\frac{1+i_t^{\mathcal{L}}}{1+i_t} = ab\mathcal{L}_t^{b-1}. \tag{4.2}$$

We proceed to take logs and subtract the resulting equation from its lagged counterpart,

$$s_t - s_{t-1} = (b-1) \cdot [\ln(\mathcal{L}_t) - \ln(\mathcal{L}_{t-1})], \tag{4.3}$$

where $s_t \approx i_t^{\mathcal{L}} - i_t$. As described above, we construct a time series for $\mathcal{L}_t$ using equation (3.1). Similarly, we measure $i_t^{\mathcal{L}}$ based on equation (3.8) using a historic deposit rate series (as described in footnote 9). We estimate (4.3) for the sample 2000:M1-2020:M4, which is the maximum time span across all data series, and obtain $b = 1.07$.

Finally, the parameters a and q inside the liquidity cost function (Φ) are selected to match a liquidity-over-GDP ratio $\mathcal{L}/Y$ of 2.4 at the quarterly frequency, and the relationship that $\Phi(\cdot) = m + d + cbdc$ in steady state, yielding $a = 0.8764$ and $q = -0.1615$.

4.3 Loan Parameters

Next, we turn to parameters related to the loan side of the model. The parameter ψ governs the importance of pledgeable capital for aggregate capital in (3.12) and therefore pins down the share of bank borrowing. Crouzet (2021) shows that this share has declined to around 30% for the most recent years, and we calibrate ψ accordingly.

For the costs of bank and bond borrowing, we obtain estimates from Schwert (2020) who compares bank loan rates and secondary bond quotes for the same firms on the same date, finding that loan and bond spreads are similar for investment-grade firms. However, estimates suggest that the average bond-implied loan spread should be around 50% of the average all-in-drawn spread of 2.8% since loans are less risky due to higher recovery rates in bankruptcy. Schwert (2020) attributes the remaining premium to banks' loan market power. To match this evidence, we equalize bond and loan spreads in steady state, that is, $\varrho = i^l - i = 2.8\%$ annually. However, banks face half of the costs of issuing credit compared with the bond market, resulting in $\varrho = 0.7\%$ for the costs of issuing bonds and $\mu^l = 0.35\%$ for the costs of issuing loans, both at the quarterly frequency.

Based on equations (3.20) and (3.21), the equivalence between bond and loan spreads in steady state implies the following relationship between φ^l and θ^k :

$$n(i+\varrho+\delta) \quad = \quad \left[(n-1)\varphi^l + \theta^k - \psi\left(\theta^k - \frac{1}{1-\alpha}\right)\right](\varrho - \mu^l), \tag{4.4}$$

where all other parameters apart from φ^l and θ^k are pinned down. Therefore, we can interpret the elasticity of substitution between bonds and loans, θ^k, as the remaining free parameter, and, conditional on that, back out φ^l from (4.4). While we lack an empirical target to pin down θ^k exactly, the model implies that it must lie in a feasible region between 1 and 11.8.[15] For our baseline specification, we choose $\theta^k = 5$ as a value in the middle of the feasible set, and show robustness to alternative values in Appendix C.2.[16]

4.4 Other Bank Parameters

Besides parameters related to the loan and deposit sides, a few other bank-related ones remain. The function for the cost of deviating from the target loan-to-equity ratio is parameterized as: $\Psi(L/F) = \kappa\nu L/F\,(\ln(L/F) - \ln\nu - 1) + \kappa\nu^2$, following Ulate (2021). The loan-to-equity target ratio, ν, and the cost of deviating, κ, are also taken from that paper, matching a steady-state loan-to-equity ratio of 9 and using cross-sectional relations between loan rates, loan amounts, and bank capital to obtain a value of κ of 12 basis points. We check the robustness of our results across values of κ in Appendix C.2.

A bank's managerial cost, ς, helps determine its profitability. Using Call Report data for commercial banks over 1984:Q1-2022:Q3, we find an average annualized return on assets close to 1%. Given the loan-to-equity ratio of 9, this implies a quarterly return-on-equity of 2.25% and we calibrate ς to match this in steady state. The fraction of bank profits staying within the bank (not paid out as dividends), ω, is calibrated such that, in the initial pre-CBDC steady state, the loan-to-equity ratio L/F is equal to its target ν.

4.5 Loan and Deposit Spreads

To provide some intuition for the behavior of spreads in the calibrated model, Figure 4.1 displays the loan rate and the deposit rate for different levels of the policy rate (which is also shown as the 45-degree line). The loan spread ranges between 2.3% and 3.5%. It is larger for higher levels of the policy rate. That is because banks gain market power at higher policy rates, raising their profitability and market share relative to bonds and increasing the endogenous loan elasticity and therefore their loan markup over the policy rate. We can see this mechanism from equations (3.19)-(3.21).

[15] The lower bound of one comes from the assumption that pledgeable and non-pledgeable capital are substitutes instead of complements. The upper bound for θ^k is φ^l, due to the nested-CES structure of the model. Given our remaining calibration and equation (4.4), this implies an upper bound for θ^k of 11.8.

[16] Based on a similar model structure, Buchak et al. (2024) estimate $\theta^k = 3.87$ which lies in the possible range we consider and is close to our baseline value of $\theta^k = 5$, providing an external validation of our choice.

The deposit rate is below the policy rate for all positive values, but is close to the policy rate for rates below -1%. The deposit spread rises with higher levels of the policy rate and bank market power. However, this relation is nonlinear. For policy rates between -1% and 5%, the deposit spread widens substantially as the policy rate increases, as targeted by our calibration strategy over these values (the yellow dots in Figure 4.1 denote our calibration targets for the deposit rate as a function of the policy rate). The widening of the deposit spread becomes smaller when the policy rate is above 5% and stabilizes at higher policy rates. This behavior of the deposit spread is consistent with the data, even though we do not target deposit rates for such high levels of the policy rate in our calibration. Thus, this provides an external validation for the empirical fit of the model.[17] To the best of our knowledge, this is the first paper that matches the empirical nonlinear dependence of the deposit spread on the policy rate. This feature is particularly relevant for the remainder of the paper and our key results, as the welfare gains of introducing CBDC are related to how bank market power in deposits is impacted across levels of the policy rate.

Figure 4.1: This figure shows the loan rate (dash-dot orange line) and the deposit rate (dashed yellow line) obtained in the baseline calibration of the model as a function of the policy rate. The policy rate is also plotted as the 45-degree line for comparison (solid blue line). The yellow dots denote the calibration targets for the deposit rate as a function of the policy rate.

[17] Such a behavior of deposit rates is reminiscent of deposit betas that are not constant but rise with higher market rates, as documented in Greenwald et al. (2023), for example. Appendix A.2 provides further details on the behavior of the pass-through of the policy rate to the deposit rate in our model.

5 Implications of CBDC Introduction

In this section, we discuss the implications of CBDC introduction through the lens of our full calibrated DSGE model. First, we focus on comparing how the economy differs between an initial steady state where CBDC is not used and a final steady state where CBDC is available, while considering various remuneration schedules for CBDC. Throughout this section, we frequently refer to the "welfare change" from introducing a CBDC which is formally the multiplicative consumption-equivalent variation, in percent, required to keep the representative household indifferent between the pre-CBDC and the post-CBDC steady state (see Appendix B.8 for details). Second, we also discuss how the economy responds to shocks around the pre-CBDC and various post-CBDC steady states. Appendix C.3 discusses the transition between steady states.

5.1 CBDC Introduction for Different CBDC Rates

We first focus on our baseline calibration, where the steady-state policy rate is 2%, and analyze outcomes of CBDC introduction for different levels of the interest rate paid on CBDC. Figure 5.1 shows the welfare change from CBDC introduction, the deposit-to-GDP ratio, and the CBDC-to-GDP ratio across CBDC rates between -1% and 3% annually. As the rate paid on CBDC increases, the CBDC-to-GDP ratio rises and the deposit-to-GDP ratio decreases monotonically. Intuitively, as the CBDC interest rate becomes more negative, the CBDC-to-GDP ratio tends to zero, since households do not want to use a very unattractive instrument. In the limit, when the CBDC rate is -100% quarterly, households do not use CBDC at all, which corresponds to our pre-CBDC scenario.

Importantly, the welfare change from CBDC introduction displays an inverted U-shape with respect to the interest rate paid on CBDC. It tends to zero when the CBDC rate approaches -100%, becomes negative for very high CBDC rates, and achieves a positive maximum of approximately 27 basis points (of initial steady-state consumption) when the CBDC rate is approximately 0.8% per year. This welfare gain is higher than the one of approximately 22 basis points when the CBDC rate is 0%, an often-discussed remuneration level by central banks that consider introducing a CBDC. Interestingly, the welfare-maximizing CBDC interest rate of approximately 0.8% per year is very close to the deposit rate in the pre-CBDC steady state.

The impact of CBDC on welfare in our model depends on three different channels. First, a CBDC can curtail commercial bank monopoly power and thereby increase the deposit rate. Second, households like some of the characteristics that CBDC has to offer. For example, a CBDC can be used for electronic transactions while it is also a direct liabil-

ity of the central bank and therefore fully insured. Such benefits are jointly captured in the model with a positive γ_{CBDC} (which is "present" even in the pre-CBDC steady state). Households therefore benefit when a CBDC is introduced because it allows them to better distribute their usage across the available liquid instruments. Third, despite a higher deposit rate, some deposits flow out of the banking system when a CBDC is introduced. The higher deposit rate and the reduced amount of deposits imply that bank equity declines, which in turn reduces credit supply, raises the cost of capital for firms, and lowers welfare. For an in-depth discussion of the intuition of these results, see Section 2.

For low and moderate levels of the CBDC rate, the first two channels described in the previous paragraph dominate the third one, leading to an increase in overall welfare due to CBDC introduction. However, for high levels of the CBDC rate, the bank disintermediation channel dominates, leading to a fall in overall welfare as observed in the right tail of the blue line in Figure 5.1. Section 6.1 provides a full decomposition of the quantitative importance of the three channels, a novel contribution of our paper.

While in our static model bank margins fall after CBDC introduction, in the full model bank return on equity—which has a component associated with deposits and another one associated with loans—is the same in the pre- and post-CBDC steady states, regardless

Figure 5.1: This figure displays some important variables for different levels of the CBDC interest rate. The welfare change (gain if positive, loss if negative) from CBDC introduction, in basis points, is in blue on the left axis, the deposit-to-GDP ratio (orange dash-dot) and the CBDC-to-GDP ratio (yellow dashed) are on the right axis.

of the CBDC remuneration schedule, as shown in Appendix B.5.1. Immediately after CBDC is introduced, deposit margins decline while loan margins are roughly unchanged, lowering overall bank returns. In turn, this pushes down bank equity, as less resources stay within the bank period-by-period. Lower bank intermediation capacity moves the banking sector up a downward sloping aggregate loan demand curve, and banks start charging slightly higher loan rates. In the limit (i.e., the post-CBDC steady state), bank equity falls enough as to make the decrease in deposit margins be offset by an increase in loan margins, leaving overall return on equity unchanged. This is a realistic feature of the model, as bank returns should not diverge from economy-wide returns in the long run. Therefore, while our model does not explicitly allow for some changes in the structure of the banking sector that may be effected by CBDC introduction (for example, bank entry or exit), it has another way of incorporating these types of realistic adjustment patterns through changes in the overall size of the banking sector.[18]

In Appendix C.2, we discuss the ratio of changes in bank credit to deposit losses in the aggregate banking sector due to the introduction of CBDC in our model. We elaborate on how this ratio depends on the elasticity of substitution between bank and nonbank borrowing, θ^k. Furthermore, we provide evidence that this ratio is not necessarily a good measure of the welfare implications of introducing a CBDC.

5.2 CBDC Introduction for Different Policy Rates

Next, we change the nature of the exercise that we perform. Instead of analyzing CBDC introduction for a given steady-state policy rate but different levels of the interest rate on CBDC, we introduce a zero-remuneration CBDC for a continuum of economies that differ in their steady-state level of the policy rate. We achieve the different steady-state levels of the policy rate by recalibrating the discount factor β, while keeping the rest of the parameters of our baseline calibration constant (however, our results are robust to recalibrating a larger set of parameters as done in Appendix C.4).[19]

Figure 5.2 illustrates how several outcome variables of interest behave for steady-state policy rates between -2% and 8% annually. As in Figure 5.1, we consider the welfare

[18] The stability of equity returns across steady states also implies that the particular assumptions governing the dividend process and bank equity accumulation in our model are not crucial for our findings. Even if hypothetical equity investors had the option to invest in banks (i.e., bank "recapitalization"), they would not choose to invest more in steady state since that would lower their expected return.

[19] Notice that the most important parameters in our model, namely the deposit-side banking parameters φ^d, θ, n, and μ^d, are calibrated to match deposit rates across levels of the policy rate. Therefore, these parameters do not need to be recalibrated when the discount factor is changed. Appendix C.4 shows that our results in this subsection and the next are robust to recalibrating additional parameters such that certain targets continue to be matched across different levels of the policy rate.

Figure 5.2: This figure displays some important variables for different levels of the policy rate. The welfare change (gain if positive, loss if negative) from CBDC introduction, in basis points, is in blue on the left axis, the deposit-to-GDP ratio (orange dash-dot) and the CBDC-to-GDP ratio (yellow dashed) are on the right axis.

change from CBDC introduction, the CBDC-to-GDP ratio, and the deposit-to-GDP ratio. As the steady-state policy rate increases, the CBDC-to-GDP ratio decreases monotonically, whereas the deposit-to-GDP ratio increases monotonically. Additionally, for high levels of the policy rate the CBDC-to-GDP ratio tends to zero, as households do not want to use a liquidity-providing instrument that pays relatively little compared to deposits. The welfare gains from CBDC introduction have an approximately monotonic behavior: they roughly fall with the steady-state policy rate and tend to zero as the policy rate rises, precisely because CBDC is mostly unused in such a scenario.

5.3 Welfare-Maximizing CBDC Rate across Policy Rates

In Section 5.1, we showed that, for our baseline steady-state policy rate of 2%, the welfare-maximizing level of the CBDC interest rate is around 0.8% per year. However, the effects of introducing a CBDC with a constant remuneration also vary substantially depending on the steady-state level of the policy rate, as shown in Section 5.2. Therefore, a natural question that emerges is: what is the CBDC interest rate that maximizes the welfare change of introducing CBDC for each level of the steady-state policy rate? Figure 5.3 displays the answer to this question. In orange, the policy rate is shown as the 45-degree

line, and in blue, the welfare-maximizing CBDC rate is plotted.

Starting on the left, for negative levels of the policy rate, the welfare-maximizing CBDC rate is negative and above the policy rate. The two cross at around -40 basis points annually. Subsequently, the welfare-maximizing CBDC rate is below the policy rate by roughly 1% annually. This welfare-maximizing CBDC rate as a function of the steady-state policy rate can be approximated fairly well by a rule-of-thumb CBDC rate that is the maximum between 0% and the policy rate minus 1%, as illustrated by the yellow line in Figure 5.3. While this approximate welfare-maximizing CBDC rate does not capture all the intricacies of the full welfare-maximizing rate, it is a rule of thumb that could easily be communicated by central banks and, in welfare terms, does almost as well as the welfare-maximizing rate, as shown below. This rule also has the benefit of avoiding negative rates on CBDC, which present a political economy concern for central banks due to the fear of the public that CBDC would be used to "expropriate their savings" with below-zero rates.

What is the intuition for the fact that the welfare-maximizing CBDC rate increases with the policy rate? The higher the policy rate, the higher the CBDC rate needs to be to take a given share of the liquid-instruments market and therefore to curtail commercial-bank market power by a given amount. To provide further intuition on this point, Figure 5.4 plots the deposit spread in the top row and the endogenous share of deposits in the

Figure 5.3: This figure displays the policy rate in orange (in both axes, so it is the 45-degree line), the welfare-maximizing level of the CBDC rate in blue, and an approximate welfare-maximizing rule-of-thumb rate, which is the maximum between 0 and the policy rate minus 1%, in yellow.

bottom row—before and after the introduction of CBDC—across levels of the policy rate (on the x-axis) and for different CBDC remuneration schedules. In the left column, we present CBDCs that pay a constant interest rate, while in the right column, we present a CBDC that pays the policy rate, a CBDC that pays the welfare-maximizing CBDC rate, and a CBDC that pays the approximately welfare-maximizing rule-of-thumb rate.

Importantly, for levels of the policy rate that are roughly above 2% per year, the welfare-maximizing CBDC rate achieves a stabilization of the deposit spread at around 70 basis points. Similarly, the endogenous deposit share is stabilized at around 65%. By contrast, a CBDC that pays a constant interest rate (regardless of the policy rate), can neither stabilize the deposit spread nor the deposit share, as visible from the left column

Figure 5.4: This figure shows the deposit spread (in the top row) and the endogenous share of deposits (bottom row), before and after the introduction of CBDC, across levels of the policy rate, for different CBDC remuneration schedules. In the left column, we present CBDCs that pay a constant interest rate, while the right column shows a CBDC that pays the policy rate, a CBDC that pays the welfare-maximizing CBDC interest rate for each level of the policy rate, and a CBDC that pays the approximately welfare-maximizing rule-of-thumb (denoted "rot") rate described in Figure 5.3.

of Figure 5.4. On the other hand, a CBDC that pays the policy rate reduces the deposit spread and the endogenous deposit share by too much relative to the welfare optimum.

Figure 5.5 plots the welfare change from introducing CBDC across different levels of the policy rate (on the x-axis). The different lines represent the alternative CBDC remuneration schedules considered in Figure 5.4. As expected, the welfare change from the welfare-maximizing CBDC rate is the envelope of the other lines. Notably, while we find modest welfare changes of around 0.25% for our baseline calibration with a policy rate of 2%, we obtain substantially higher welfare changes of 1% and beyond if the policy rate is around 6% or higher. As mentioned above, the welfare change associated with the rule-of-thumb CBDC rate is almost identical to the one of the welfare-maximizing rate.

5.4 Responses to Monetary Policy Shocks

Having already examined how the economy reacts to the introduction of CBDC by comparing the initial pre-CBDC steady state with the final post-CBDC steady state, we now turn to analyzing how the two economies differ in their response to transitory shocks around the respective steady states. We focus on the impulse responses to a monetary policy shock in this section, but our main findings also apply to technology shocks, as illustrated in Appendix C.5.

Figure 5.5: This figure shows the welfare change from the introduction of CBDC, in percent, across levels of the policy rate (in the x-axis), for different CBDC remuneration schedules described in Figure 5.4.

Figure 5.6 depicts the impulse responses of several important variables to a 50 basis point expansionary monetary policy shock for different CBDC remuneration schedules. The dotted black line shows the pre-CBDC case, which we compare to the following cases: (i) a CBDC that pays a constant interest rate of 0%, (ii) a CBDC that pays a constant rate at the welfare-maximizing level (of roughly 0.8% annually) for a policy rate of 2%, (iii) a CBDC that pays the policy rate minus 1%, corresponding to the approximated rule-of-thumb CBDC rate, and (iv) a CBDC that pays the policy rate.

Even though these regimes have significantly different welfare implications, the impulse responses are remarkably similar. To understand why the IRFs to transitory shocks do not change much with CBDC introduction or its remuneration schedule, it is useful to

Figure 5.6: This figure depicts the impulse response functions to a 50 basis point expansionary monetary policy shock for different CBDC remuneration schedules. The line denoted $i^{CBDC} = \overline{opt}$ stands for a CBDC that pays a rate that is constant at the welfare-maximizing level for a steady-state policy rate of 2%, which is 0.8% per year.

discuss the two main neoclassical channels of interest rate transmission in our model: the intertemporal substitution channel and the investment-based channels.

Since the pass-through of the policy rate to the deposit rate improves after the introduction of CBDC, one could think that the intertemporal substitution channel could be stronger after CBDC introduction, particularly if the CBDC rate varies with the policy rate. However, the important thing to note here is that households in our model are allowed to save in bonds (although bonds do not provide liquidity benefits). Therefore, the relevant interest rate governing the consumption-saving trade-off in our model's Euler equation, expression (3.4), is the bond rate, which coincides with the policy rate. Since the pass-through of the policy rate to the interest rate governing the consumption-saving decision is already perfect, it cannot be improved by the introduction of CBDC.[20]

The investment-based channels also do not get substantially altered with CBDC introduction. This is due to the fact that transitory shocks have a limited impact on bank equity and lending capacity.[21] Therefore, these shocks do not lead to major differences in investment and capital accumulation from interactions between the banking system and the existence of CBDC or its remuneration schedule. To establish this in a scenario where bank lending mechanisms are given the best possible chance of "making a difference", we perform an alternative calibration where the importance of banks in overall lending is much higher than in our baseline calibration (as in the European calibration we discuss in Section 6.2, where $\psi = 82\%$), and where the substitution between bank lending and bond borrowing is lower ($\theta^k = 1.1$). Even in such an economy where bank lending is substantially less dispensable than in our baseline, the responses to transitory monetary policy shocks are still very similar across the different CBDC scenarios.

6 Discussion

In this section, we discuss additional issues that are relevant to paint a richer picture of the potential impacts of CBDC. First, we decompose the welfare impacts of CBDC into three

[20] In an alternative model where a fraction of households were only allowed to save in the liquid instruments, the pass-through of the policy rate to the rate on these instruments would become more relevant for the intertemporal substitution channel and the IRFs to monetary policy shocks are more likely to be impacted by the introduction of CBDC. Importantly, even in such an extended model, relatively high-wealth agents would drive the bulk of the consumption-saving margin, and in the "real world" these high-wealth agents certainly have access to financial instruments that move closely with the policy rate.

[21] Even though bank profitability and equity evolve differently across different CBDC remuneration scenarios, a 50 basis points monetary policy shock leads at most to a 2% deviation in bank equity from its steady-state value across a range of CBDC remuneration scenarios and values for the importance of banks in lending (ψ) and the elasticity of substitution between pledgeable and non-pledgeable capital (θ^k).

channels: liquidity benefits, curtailing bank monopoly power in deposits, and the bank disintermediation effect. Second, we perform an alternative calibration for the Euro Area. Third, we consider potential costs of issuing or operating a CBDC and how they can affect the welfare analysis. Fourth, we explore a link between bank reserves and wholesale or interbank funding in an extended framework where banks have access to such alternative funding sources. Fifth, we discuss bank taxes or subsidies as an alternative to curtailing bank monopoly power and the viability (or lack thereof) of this strategy. Finally, we briefly touch upon CBDC holding limits.

6.1 Decomposing the Welfare Impacts of CBDC

There are three key channels driving the welfare effects of CBDC. First, introducing CBDC affects the amount of liquidity services that households obtain through the $\Phi(\cdot)$ function. Second, introducing CBDC can curtail commercial bank deposit market power. Third, CBDC introduction can produce a bank disintermediation effect. So far, our welfare analysis combined all of these channels. In this section, we decompose the welfare impact to better understand the quantitative importance of each channel.

Starting from the full model, we can shut down the bank disintermediation effect simply by setting the parameter κ, which determines the importance of bank equity for lending, to zero. The orange dash dotted line in Figure 6.1 depicts the welfare effect of introducing CBDC in an alternative economy where $\kappa = 0$. This economy only incorporates the effects of the liquidity services and the curtailing of commercial bank market power in deposits (we therefore denote it "Liq+MPD", for "Liquidity" and "Market Power in Deposits"). When $\kappa = 0$, bank equity is irrelevant for lending, and the bank disintermediation effect, which reduces the welfare gains of CBDC, is therefore absent. As a result, the orange line is substantially higher than the blue solid line, which contains all three channels and replicates the blue solid line in Figure 5.1. The difference between the orange dash dotted line and the blue solid line can be interpreted as the welfare impact of introducing CBDC solely through the bank disintermediation channel, and is given by the purple line with circular markers in Figure 6.1.[22]

Starting from the economy with $\kappa = 0$, we can further make the deposit market fully competitive by tending the number of banks (n) and the elasticity of substitution between banks in deposits (φ^d) to infinity.[23] The yellow dashed line in Figure 6.1 depicts the wel-

[22]While the pre-CBDC steady states are different between the economy with $\kappa = 0$ and our baseline with $\kappa > 0$, the differences for key variables (like consumption, labor, output, etc.) are generally smaller than 1%. Thus, differences in the welfare effects across these various steady states are still informative.

[23]We also set the banking costs ς, μ^d, and μ^l to zero and the elasticity of substitution between banks in loans

fare effect of introducing CBDC in this alternative economy. With a perfectly competitive banking sector, the economy only incorporates the effects of the liquidity services channel (we therefore simply denote it "Liq") captured by the $\Phi(\cdot)$ function. The benefits of curtailing commercial bank market power are no longer present, and therefore the yellow dashed line is below the orange dashed-dotted line for all levels of the CBDC rate. The difference between the orange dashed-dotted line and the yellow dashed line, which can be interpreted as the welfare impact of introducing CBDC just through the curtailment of commercial bank market power, is given by the black dotted line in Figure 6.1.

Besides the level differences, Figure 6.1 also shows how the welfare impacts of the various channels differ across CBDC rates. As expected, the disintermediation channel

Figure 6.1: This figure decomposes the welfare gain from issuing CBDC for the baseline calibrated economy across three channels: increased access to liquid savings instruments (denoted "Liq" and depicted by the yellow dashed line), the effects of curtailing commercial bank market power in deposits (denoted "MPD" and depicted by the black dotted line), and the effects of disintermediating commercial bank lending activity (denoted "Dis" and depicted by the purple line with circular markers). Combinations of these channels are depicted by the orange dashed-dotted line ("Liq + MPD") and the blue solid line ("Liq + MPD + Dis") which contains all three channels present in the baseline model and therefore replicates the solid blue line in Figure 5.1. The main text contains details on how the decomposition across channels is performed.

(φ^l) to infinity to achieve a fully frictionless banking sector. The extra cost of borrowing in corporate bonds (ϱ) is also set to zero, and the fraction of end-of-period-resources staying in the bank (ω) is chosen to ensure a well-defined steady state where $L/F = \nu$. In this economy, the deposit rate, the loan rate, and the cost of borrowing in corporate bonds all equal the policy rate (i.e., $i^d = i^l = i + \varrho = i$).

leads to negative welfare effects of introducing CBDC. These negative impacts increase with the CBDC rate, as a higher fraction of liquidity moves from deposits to CBDC, reducing commercial bank profits and lending capacity. The effects of introducing CBDC through the liquidity channel display an inverted U-shape, with a maximum around a CBDC rate of 2%. The decreasing portion to the right of 2% occurs because CBDC holdings are being inefficiently subsidized as the CBDC rate goes far above the policy rate. Finally, the effects of introducing CBDC just from the curtailment of commercial bank monopoly power increase with the CBDC rate. For the welfare-maximizing CBDC rate of 0.8% per year, the welfare gains from the liquidity channel are 22 basis points, those from curtailing bank deposit market power are 38 bps, and those from the disintermediation channel are -33 bps, indicating that (in absolute value) all three channels are of roughly similar importance for our results.

6.2 Euro-Area Calibration

Our main calibration so far has focused on the U.S. However, the Euro Area is more advanced in its research of CBDC. Additionally, the economy of the Euro Area has some characteristics that differ from those in the U.S., particularly regarding the importance of commercial banks for total lending. Thus, we perform an alternative calibration for the Euro Area. While several calibrated parameters change slightly, as detailed in Appendix C.6, the key difference is an increase in the importance of pledgeable capital, ψ, from 30% to 82%. This change amplifies the importance of the bank disintermediation channel since commercial banks account for a larger fraction of overall lending and reducing their lending capacity has a more substantial negative impact on welfare. As a consequence, while the welfare gains for the U.S. peak at 26.8 basis points for a CBDC rate of 84 basis points, the welfare gains for Europe peak at 17.8 basis points for a CBDC rate of 64 basis points. The Euro Area can therefore achieve a maximum of 9 basis points lower welfare gain than the U.S., and this happens at a CBDC rate that is 20 basis points lower.

6.3 Costs of Issuing and Operating a CBDC

Our model has the flexibility to incorporate the per-period costs of operating a CBDC system. While our baseline sets this cost to zero for simplicity, in this section we discuss how these costs may alter the analysis, focusing on the Euro Area where some information is available about these potential costs. We gather three different types of CBDC costs. First, the one-time costs for commercial banks of adjusting to the new CBDC system. These costs have been estimated at around 18 billion euros for the entire Euro Area, while

the European Commission estimates them at a lower level of around 5.4 billion.[24] We use 20 billion euros to be as conservative as possible. The second type of costs are the internal costs to the ECB of setting up the CBDC system. While the initial vendor calls for some components of the CBDC system are around 1.1 billion euros, that does not include the core CBDC settlement system, which is expected to be "sourced internally" at the ECB. We use a number of 30 billion here as an upper estimate as well. The third type of costs are the annual operating expenses of the system. While this number has not been estimated in a credible way to the best of our knowledge, we use 800 million annually as an upper bound, representing over 50% of the total 2024 operating expenses of the ECB.

Using a quarterly interest rate of 0.5%, we convert the 50 billion one-time cost into a quarterly expense of 250 million, resulting in a total quarterly cost of 450 million euros. This represents less than 2.2 basis points of quarterly Euro Area consumption in 2025. Given the potential welfare gain from CBDC introduction of 17.8 basis points of initial steady-state consumption that we find for the Euro Area, the costs of issuing and operating the CBDC system would represent less than 12% of the overall welfare gain, a relatively small share that does not substantially alter our main conclusion. For the United States, where our estimated welfare gains from CBDC introduction are higher, the CBDC issuance and operating costs would likely represent an even smaller share of the gains for equivalent cost estimates (as a share of steady-state consumption or GDP).

6.4 Disintermediation, Reserves, and Wholesale or Interbank Funding

So far, we have referred to the key negative consequence of introducing CBDC as a "Bank Disintermediation" channel. However, it is important to notice that banks in our model are free to substitute lost deposit funding with borrowing from the central bank at the policy rate. Therefore, the "disintermediation" does not take place because banks do not have enough funds to lend, it happens instead because deposit funding earns a spread that contributes positively to bank profitability, while funds borrowed from the central bank do not. If the central bank were to compensate commercial banks for this lost spread after the introduction of CBDC, as in Brunnermeier and Niepelt (2019), this channel could be mitigated or eliminated. However, such a subsidization of banks is likely politically untenable.

We also note that the representative bank in the post-CBDC steady state is a net borrower from the central bank (i.e., $\overline{H}_j < 0$). At first glance, this may be considered unre-

[24] The sources for these estimates and the following ones are a PWC cost study, news reports about the European Commission's study and ECB's vendor calls, as well as the ECB's expense report.

alistic. However, this outcome stems from the fact that we did not explicitly allow banks to access wholesale funding. If banks had such funding available at the policy rate, then our post-CBDC steady state can be expressed instead as banks having a positive amount of wholesale funding and positive excess reserves simultaneously, with no change to our results.[25] Likewise, introducing an interbank market with a rate that is located somewhere between the policy rate and the policy rate plus a small spread (typically of 25 basis points) leaves our results both qualitatively and quantitatively unchanged.

6.5 Bank Taxes and Subsidies

One of the positive effects of introducing CBDC in our model is to curtail commercial bank market power in deposits. We acknowledge that there are alternative tools to address this issue, namely taxes or subsidies on deposits. Importantly, just using taxes or subsidies to curtail commercial bank market power does not achieve the liquidity benefits of introducing CBDC, lowering the overall attainable welfare gains. Additionally, the tax/subsidy remedy for commercial bank market power in deposits entails *subsidizing* commercial banks (not *taxing* them) which could be politically unpopular, as banks are not necessarily well regarded among the American or European public.[26] Another issue arises in the case of heterogeneous banks. The subsidies required to address market power in deposits would vary by bank and would require the central bank to have a lot of information about individual banks to set the correct subsidy. By contrast, CBDC carries a single interest rate, requiring less information while addressing market power in deposits in a way that automatically varies across banks depending on their market power.

6.6 Holding Limits

Several central banks are coalescing around imposing holding limits on CBDC. In our model, the introduction of CBDC leads to positive welfare effects for a wide range of CBDC interest rates, even without holding limits. Adopting binding holding limits has the potential to reduce the ability of a CBDC to curtail commercial bank market power

[25] To illustrate this point, consider the expression for the bank balance sheet divided by bank equity: $L/F + H/F = 1 + D/F$. In the post-CBDC steady state, these quantities take the values $L/F \approx 9.25$, $H/F \approx -1.6$, and $D/F \approx 6.65$, which indicates that banks borrow a substantial amount from the discount window. However, if we extend the balance sheet to $L/F + \tilde{H}/F = 1 + D/F + W/F$, where W represents wholesale funding, then the same values for the loan and deposit ratios in the post-CBDC steady state can be supported by $\tilde{H}/F \approx 0.4$ and $W/F \approx 2$, for example, so that banks hold excess reserves but borrow from the wholesale market. Since the household's supply of funds at the policy rate is perfectly elastic, changing the amount that banks demand makes no difference to our results.

[26] See, e.g., this survey reported on by the American Bankers Association.

in deposits and may reduce its overall welfare benefits. To assess whether some commonly proposed holding limits might be binding in our model, we perform a back-of-the-envelope calculation. Quarterly U.S. GDP per capita in 2024 was around 21 thousand dollars, so a limit of three thousand dollars, if fully utilized by the population, would imply a CBDC-to-GDP ratio of around 15%. As illustrated in Figure 5.1, the CBDC-to-GDP ratio in our model is around 32% when the CBDC interest rate is 0%, and reaches 64% for the welfare maximizing CBDC interest rate of 0.8%. Therefore, holding limits of the considered amount would be insufficient to achieve the maximum benefits in our model.

That being said, holdings limits may be beneficial in times of financial distress when uninsured depositors may withdraw funds from banks and convert them to CBDC, which may in turn exacerbate financial turmoil. While our model abstracts from the impact of CBDC on financial stability, a line of research that studies this relation often builds on the seminal work of Diamond and Dybvig (1983), and includes Fernandez-Villaverde et al. (2021), Keister and Monnet (2022), and Bidder et al. (2024), though these models exclude bank market power. A salient path for future research would be to integrate financial crises into our macroeconomic framework with endogenous bank market power to study how the introduction and remuneration of CBDC might affect the frequency and severity of crises, as well as the role of holding limits in this regard.

7 Conclusion

In this paper, we develop a macroeconomic model to assess the introduction of a CBDC. Three competing channels determine the welfare effects in our model. On the positive side, households benefit from the introduction of a CBDC in two ways. First, they value the expansion of liquidity services that the new saving instrument provides. Second, households receive higher deposit rates since CBDC competes with bank deposits, thus reducing banks' deposit market power. On the negative side, banks face deposit outflows and cut their lending, which in turn reduces aggregate investment and output.

We assess this welfare trade-off for a wide range of economies that differ in their level of interest rates. We find substantial welfare benefits from introducing CBDC if countries follow a simple rule that determines the rate of interest on CBDC as the maximum between 0% and the policy rate minus 1%. The simplicity of this rule is appealing and avoids political-economy concerns related to paying negative rates on CBDC. Interestingly, we also find that the introduction of a CBDC is most beneficial for economies with high interest rates. In such environments, banks have substantial market power in deposit markets which is sharply curtailed once a CBDC is introduced.

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Online Appendix for

A Macroeconomic Model of Central Bank Digital Currency

by Pascal Paul, Mauricio Ulate, and Jing Cynthia Wu

Appendix A Solving the Static Bank Model

First, substitute the balance sheet condition (2.5) into the objective function and write d_j as an implicit function of $1+i_j^d$, then the bank's problem becomes

$$\max_{i_j^d} \qquad (1+i)(f_j+d_j)-(1+i_j^d)d_j,$$

Take the first-order condition with respect to $1+i_j^d$

$$-d_j+\left((1+i)-(1+i_j^d)\right)\epsilon_j^d\frac{d_j}{1+i_j^d}=0,$$

where $\epsilon_j^d \equiv \partial \ln d_j/\partial \ln(1+i_j^d)$. Rearranging this equation we can obtain (2.6). Next, use (2.3) to express ϵ_j^d as:

$$\begin{aligned}\epsilon_j^d &= \frac{\partial d_j}{\partial(1+i_j^d)}\frac{1+i_j^d}{d_j}\\ &= \varphi^d-\varphi^d\frac{\partial\ln(1+i^d)}{\partial\ln(1+i_j^d)}+\frac{\partial\ln d}{\partial\ln(1+i^d)}\frac{\partial\ln(1+i^d)}{\partial\ln(1+i_j^d)}.\end{aligned} \tag{A.1}$$

Define the elasticity of the aggregate gross deposit rate w.r.t. an individual gross deposit rate and use (2.4) and (2.3) to write it as:

$$\omega_d^{d_j} \equiv \frac{\partial\ln(1+i^d)}{\partial\ln(1+i_j^d)}=\frac{1}{n}\left(\frac{1+i_j^d}{1+i^d}\right)^{\varphi^d+1}=\frac{(1+i_j^d)d_j}{(1+i^d)d}. \tag{A.2}$$

We can interpret this as the share of overall deposits that are maintained at bank j. Using (2.1), the elasticity of aggregate deposits w.r.t the gross deposit rate can be expressed as:

$$\frac{\partial\ln d}{\partial\ln(1+i^d)} = \theta\left(1-\frac{\partial\ln(1+i^{\mathcal{L}})}{\partial\ln(1+i^d)}\right)\equiv\theta\left(1-\omega_{\mathcal{L}}^d\right), \tag{A.3}$$

where the last equality is by definition. Using (2.2) and (2.1) we can further express $\omega_{\mathcal{L}}^d$ as:

$$\omega_{\mathcal{L}}^d\equiv\frac{\partial\ln(1+i^{\mathcal{L}})}{\partial\ln(1+i^d)}=\gamma_d\left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1}=\frac{(1+i^d)d}{(1+i^{\mathcal{L}})\mathcal{L}}.$$

Finally, substitute (A.2) and (A.3) into (A.1) to obtain

$$\epsilon_j^d = (1-\omega_d^{d_j})\varphi^d + \omega_d^{d_j}(1-\omega_{\mathcal{L}}^d)\theta. \tag{A.4}$$

When all banks are identical, in a symmetric equilibrium, they all pay the same deposit rate $i_j^d = i^d$, face the same elasticity $\epsilon_j^d = \epsilon^d$, and obtain one n-th of total deposit. Consequently, $\omega_d^{d_j} = 1/n$, and we obtain equation (2.7).

Once symmetry across banks has been imposed in the model of Section 2, the equilibrium system for the determination of the endogenous deposit rate is composed of equation (2.6) for the representative bank, the definition of $\omega_{\mathcal{L}}^d$ in equation (2.8), the definition of the interest rate on liquidity in equation (2.2), as well as the equation for the behavior of the endogenous deposit markdown (2.7). Introduce the third into the second and simplify to obtain:

$$\begin{aligned}
1+i^d &= \frac{\epsilon^d}{\epsilon^d+1}(1+i) \\
\omega_{\mathcal{L}}^d &= \frac{\gamma_d}{\gamma_m\left(\frac{1}{1+i^d}\right)^{\theta+1} + \gamma_d + (1-\gamma_m-\gamma_d)\left(\frac{1+i^{cbdc}}{1+i^d}\right)^{\theta+1}} \\
\epsilon^d &= \frac{n-1}{n}\varphi^d + \frac{\theta}{n}(1-\omega_{\mathcal{L}}^d).
\end{aligned}$$

This is a system of three equations in three endogenous variables $(i^d, \omega_{\mathcal{L}}^d, \epsilon^d)$ and several exogenous variables $(i, i^{cbdc}, \gamma_m, \gamma_d, n, \varphi^d)$. The system is implicit and cannot be solved in closed form. Therefore, we apply the implicit function theorem to determine how changes in exogenous variables affect the endogenous variables. First, we show that in a special case, there is a known solution to the system that we can apply the implicit function theorem around.

Appendix A.1 Implicit Function Theorem Application

Denote with x all the exogenous variables and with y the three endogenous ones, simplify the notation of $\omega_{\mathcal{L}}^d$ to just ω, and define:

$$\begin{aligned}
F_1(x,y) &= 1+i^d - \frac{\epsilon^d}{\epsilon^d+1}(1+i) \\
F_2(x,y) &= \omega - \frac{\gamma_d}{\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d} \\
F_3(x,y) &= \epsilon^d - \frac{n-1}{n}\varphi^d - \frac{\theta}{n}(1-\omega).
\end{aligned}$$

Then we can apply the implicit function theorem to our system of equations that can be represented by $F(x,y)=0$. We can write the matrix of derivatives of the F's w.r.t. the endogenous variables as:

$$D_yF = \begin{bmatrix} \frac{\partial F_1}{\partial i^d} & \frac{\partial F_1}{\partial \omega} & \frac{\partial F_1}{\partial \epsilon^d} \\ \frac{\partial F_2}{\partial i^d} & \frac{\partial F_2}{\partial \omega} & \frac{\partial F_2}{\partial \epsilon^d} \\ \frac{\partial F_3}{\partial i^d} & \frac{\partial F_3}{\partial \omega} & \frac{\partial F_3}{\partial \epsilon^d} \end{bmatrix} = \begin{bmatrix} 1 & 0 & a \\ b & 1 & 0 \\ 0 & c & 1 \end{bmatrix},$$

where:

$$
\begin{aligned}
a &= -\frac{1+i}{(\epsilon^d+1)^2} < 0 \\
b &= -\frac{\gamma_d(1+\theta)(1+i^d)^{-\theta-2}\left[\gamma_m + (1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}\right]}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d\right)^2} < 0 \\
c &= \frac{\theta}{n} > 0.
\end{aligned}
$$

The determinant of D_yF is $1+abc$, which is positive because of the signs of a, b, and c. Moreover, we can also calculate the inverse of D_yF (using the transpose of the matrix of cofactors divided by the determinant):

$$
(D_yF)^{-1} = \frac{1}{1+abc}\begin{bmatrix} 1 & ac & -a \\ -b & 1 & ab \\ bc & -c & 1 \end{bmatrix},
$$

We can also write:

$$
D_xF = \begin{bmatrix} \frac{\partial F_1}{\partial i} & \frac{\partial F_1}{\partial i^{cbdc}} & \frac{\partial F_1}{\partial \gamma_m} & \frac{\partial F_1}{\partial \gamma_d} & \frac{\partial F_1}{\partial n} & \frac{\partial F_1}{\partial \varphi^d} \\ \frac{\partial F_2}{\partial i} & \frac{\partial F_2}{\partial i^{cbdc}} & \frac{\partial F_2}{\partial \gamma_m} & \frac{\partial F_2}{\partial \gamma_d} & \frac{\partial F_2}{\partial n} & \frac{\partial F_2}{\partial \varphi^d} \\ \frac{\partial F_3}{\partial i} & \frac{\partial F_3}{\partial i^{cbdc}} & \frac{\partial F_3}{\partial \gamma_m} & \frac{\partial F_3}{\partial \gamma_d} & \frac{\partial F_3}{\partial n} & \frac{\partial F_3}{\partial \varphi^d} \end{bmatrix} = \begin{bmatrix} -\frac{\epsilon^d}{\epsilon^d+1} & 0 & 0 & 0 & 0 & 0 \\ 0 & e & f & g & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\varphi^d-\theta+\theta\omega}{n^2} & \frac{1}{n}-1 \end{bmatrix},
$$

where:

$$
\begin{aligned}
e &= \frac{\gamma_d(1-\gamma_m-\gamma_d)(1+\theta)(1+i^{cbdc})^{\theta}(1+i^d)^{-\theta-1}}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d\right)^2} > 0 \\
f &= \frac{\gamma_d \frac{1-(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d\right)^2} \lesseqgtr 0 \\
g &= -\frac{\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d - \gamma_d\left(1-\frac{(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}\right)}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d\right)^2} \\
&= -\frac{\frac{\gamma_m+(1-\gamma_m)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}}}{\left(\frac{\gamma_m+(1-\gamma_m-\gamma_d)(1+i^{cbdc})^{\theta+1}}{(1+i^d)^{\theta+1}} + \gamma_d\right)^2} < 0.
\end{aligned}
$$

Notice that f has the opposite sign of i^{cbdc}. That is, if i^{cbdc} is positive, then f is negative, if $i^{cbdc}=0$, then $f=0$, and if i^{cbdc} is negative then f is positive. We can use the implicit function theorem to write:

$$
\begin{aligned}
D_xy &= \begin{bmatrix} \frac{\partial i^d}{\partial i} & \frac{\partial i^d}{\partial i^{cbdc}} & \frac{\partial i^d}{\partial \gamma_m} & \frac{\partial i^d}{\partial \gamma_d} & \frac{\partial i^d}{\partial n} & \frac{\partial i^d}{\partial \varphi^d} \\ \frac{\partial \omega}{\partial i} & \frac{\partial \omega}{\partial i^{cbdc}} & \frac{\partial \omega}{\partial \gamma_m} & \frac{\partial \omega}{\partial \gamma_d} & \frac{\partial \omega}{\partial n} & \frac{\partial \omega}{\partial \varphi^d} \\ \frac{\partial \epsilon^d}{\partial i} & \frac{\partial \epsilon^d}{\partial i^{cbdc}} & \frac{\partial \epsilon^d}{\partial \gamma_m} & \frac{\partial \epsilon^d}{\partial \gamma_d} & \frac{\partial \epsilon^d}{\partial n} & \frac{\partial \epsilon^d}{\partial \varphi^d} \end{bmatrix} \\
&= -(D_yF)^{-1}D_xF
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{1+abc}\begin{bmatrix} 1 & ac & -a \\ -b & 1 & ab \\ bc & -c & 1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{\epsilon^d}{\epsilon^d+1} & 0 & 0 & 0 & 0 & 0 \\ 0 & e & f & g & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\varphi^d-\theta+\theta\omega}{n^2} & \frac{1}{n}-1 \end{bmatrix} \\
&= -\frac{1}{1+abc}\begin{bmatrix} -\frac{\epsilon^d}{\epsilon^d+1} & ace & acf & acg & a\frac{\varphi^d-\theta+\theta\omega}{n^2} & a\left(1-\frac{1}{n}\right) \\ b\frac{\epsilon^d}{\epsilon^d+1} & e & f & g & -ab\frac{\varphi^d-\theta+\theta\omega}{n^2} & ab\left(\frac{1}{n}-1\right) \\ -bc\frac{\epsilon^d}{\epsilon^d+1} & -ce & -cf & -cg & -\frac{\varphi^d-\theta+\theta\omega}{n^2} & \frac{1}{n}-1 \end{bmatrix}.
\end{aligned}
$$

Knowing the sign of all the letters ($a<0, b<0, c>0, e>0, f \lesseqgtr 0, g<0$) we can sign these derivatives:

$$
\begin{bmatrix} \frac{\partial i^d}{\partial i} & \frac{\partial i^d}{\partial i^{cbdc}} & \frac{\partial i^d}{\partial \gamma_m} & \frac{\partial i^d}{\partial \gamma_d} & \frac{\partial i^d}{\partial n} & \frac{\partial i^d}{\partial \varphi^d} \\ \frac{\partial \omega}{\partial i} & \frac{\partial \omega}{\partial i^{cbdc}} & \frac{\partial \omega}{\partial \gamma_m} & \frac{\partial \omega}{\partial \gamma_d} & \frac{\partial \omega}{\partial n} & \frac{\partial \omega}{\partial \varphi^d} \\ \frac{\partial \epsilon^d}{\partial i} & \frac{\partial \epsilon^d}{\partial i^{cbdc}} & \frac{\partial \epsilon^d}{\partial \gamma_m} & \frac{\partial \epsilon^d}{\partial \gamma_d} & \frac{\partial \epsilon^d}{\partial n} & \frac{\partial \epsilon^d}{\partial \varphi^d} \end{bmatrix} = \begin{bmatrix} + & + & ? & - & + & + \\ + & - & ? & + & + & + \\ - & + & ? & - & + & + \end{bmatrix},
$$

where we required $\varphi^d > \theta(1-\omega)$ to sign the fifth column, but this requirement is less stringent than $\varphi^d > \theta$ which we should assume anyway (more substitutability in the inner nest than the outer nest, saying that banks are more substitutable with each other than deposits are substitutable with cash and CBDC). The signs of the third column are -, + , - if $i^{cbdc} > 0$, all zero if $i^{cbdc} = 0$, and +, - , + if $i^{cbdc} < 0$.

For the proof of the reaction of aggregate deposits, d, to any exogenous variable, notice that equations (2.1) and (2.8) can be combined to obtain

$$
d = \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta} \boldsymbol{\mathcal{L}} = \gamma_d \left(\frac{\omega^d_{\mathcal{L}}}{\gamma_d}\right)^{\frac{\theta}{\theta+1}} \boldsymbol{\mathcal{L}}.
$$

Since $\mathcal{L}$ is constant in this setup, and $\frac{\theta}{\theta+1} > 0$, then the sign of $\frac{\partial d}{\partial z}$ is the same as the sign of $\frac{\partial \omega^d_{\mathcal{L}}}{\partial z}$, for any exogenous variable z (other than γ_d).

Appendix A.2 Pass-Through of the Policy Rate to the Deposit Rate

In this appendix, we analyze the pass-through of the policy rate to the deposit rate and how that depends on parameters, and whether this pass-through has a minimum, what that minimum is, and how it depends on parameters. Relative to the static model in Section 2, we introduce the μ^d cost of issuing deposits that we adopt in the full model, and we also allow total liquidity to be endogenous as in the full model, denoting $\varphi^{\mathcal{L}} \equiv (\partial \ln \mathcal{L})/(\partial \ln(1+i^{\mathcal{L}}))$. To start, notice that, in the pre-CBDC scenario where $i^{cbdc} = -100\%$ and where cash pays zero percent, the deposit rate depends on just four equations (we ignore the time subscripts for notational convenience and because all variables are dated the same):

$$
(1+i^{\mathcal{L}})^{\theta+1} = \gamma_m + \gamma_d (1+i^d)^{\theta+1} \tag{A.5}
$$

$$
\omega^d_{\mathcal{L}} = \gamma_d \left(\frac{1+i^d}{1+i^{\mathcal{L}}}\right)^{\theta+1} \tag{A.6}
$$

$$
\epsilon^d = \frac{n-1}{n}\varphi^d + \frac{\theta}{n} - \frac{\omega^d_{\mathcal{L}}}{n}\left(\theta - \varphi^{\mathcal{L}}\right) \tag{A.7}
$$

$$1 + i^d = \frac{\epsilon^d}{\epsilon^d + 1}(1 + i - \mu^d). \tag{A.8}$$

We can combine all of these into a single equation in i and i^d and then use the implicit function theorem to compute the derivative of i^d w.r.t. i, and then we can see how this object (which is the pass-through) behaves. Combining (A.5)-(A.8) and simplifying we can obtain:

$$n(1 + i^d) = \left[(n-1)\varphi^d + \theta - \frac{1}{1 + \frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}}\left(\theta - \varphi^{\mathcal{L}}\right)\right](i - \mu^d - i^d). \tag{A.9}$$

This is an equation in the variables i and i^d and the parameters n, φ^d, θ, γ_m, γ_d, $\varphi^{\mathcal{L}}$, and μ^d. Now we write:

$$F(i^d, i) = n(1 + i^d) - \left[(n-1)\varphi^d + \theta - \frac{1}{1 + \frac{\gamma_m}{\gamma_d}\frac{1}{(1+i^d)^{\theta+1}}}\left(\theta - \varphi^{\mathcal{L}}\right)\right](i - \mu^d - i^d). \tag{A.10}$$

So, the equilibrium equation for i^d as a function of i can be written as:

$$F(i^d, i) = 0. \tag{A.11}$$

Then, if the assumptions of the implicit function theorem are satisfied, we know that:

$$\frac{di^d}{di} = -\frac{F_i}{F_{i^d}}. \tag{A.12}$$

Since this is the derivative of i^d w.r.t. i, it has the interpretation of the pass-through of the policy rate to the deposit rate, which is an important object in papers like Drechsler et al. (2017, 2021). Notice, also, that if we want the second derivative of i^d w.r.t. i we can also use the implicit function theorem for this:

$$\frac{d^2 i^d}{di^2} = \frac{2F_i F_{i^d} F_{ii^d} - F_{ii} F_{i^d}^2 - F_{i^d i^d} F_i^2}{F_{i^d}^3}. \tag{A.13}$$

We want to study if there is a value of the policy rate for which $\frac{d^2 i^d}{di^2} = 0$, and then we can obtain the value of the pass-through, $\frac{di^d}{di}$, at that value of the policy rate, to obtain the minimum pass-through and evaluate how it depends on parameters. After a substantial amount of algebra, we can obtain an expression for the inverse minimum pass-through as a function of just four parameter values n, φ^d, $\varphi^{\mathcal{L}}$, and θ:

$$\left(\left(\frac{di^d}{di}\right)^*\right)^{-1} = 1 + \frac{n}{(n-1)\varphi^d + \varphi^{\mathcal{L}}} + \frac{n(\theta - \varphi^{\mathcal{L}})\theta^2}{4[(n-1)\varphi^d + \theta](1+\theta)[(n-1)\varphi^d + \varphi^{\mathcal{L}}]} \tag{A.14}$$

This is an exact expression for the (inverse) minimum pass-through as a function of four relevant parameters n, φ^d, $\varphi^{\mathcal{L}}$, and θ. This tells us that the inverse minimum pass-through is always between 1 and infinity, so the minimum pass-through is always between 0 and 1. It is easy to see that when $n \to \infty$, the inverse minimum pass-through tends to $1 + \frac{1}{\varphi^d}$.

The expression for the pass-through tells us that a higher $\varphi^{\mathcal{L}}$ always increases the minimum pass-through while a higher θ decreases the minimum pass-through. Therefore, if one intended to find the

parameter values that lower the minimum pass-through, one would pick the lowest possible $\varphi^{\mathcal{L}}$ and the highest possible θ. However, we also require $0 \leq \varphi^{\mathcal{L}} \leq \theta \leq \varphi^{d}$, so in order to obtain the lowest possible minimum pass-through w.r.t. $\varphi^{\mathcal{L}}$ and θ one can pick $\varphi^{\mathcal{L}} = 0$ and $\theta = \varphi^{d}$. In this case, the expression for the inverse minimum pass-through is:

$$\left(\left(\frac{di^d}{di}\right)^*\right)^{-1} = 1 + \frac{n}{(n-1)\varphi^d} + \frac{\varphi^d}{4(1+\varphi^d)(n-1)} \tag{A.15}$$

While this expression depends both on n and φ^d, if n is too big, then there are no reasonable values of φ^d that can achieve a minimum pass-through of 50% or lower. Therefore, the model requires a low n to be able to match a low minimum pass-through.

Appendix A.3 Optimal CBDC Rate for Liquidity Provision

In this subsection, we explore what is the optimal interest rate on CBDC rate just from the perspective of liquidity management. As discussed in Section 6.1, CBDC impacts welfare through three main channels in our model, it affects the liquidity services that household receive, it affects commercial bank monopoly power in deposits, and it affects bank lending. Imagine that we abstract from the last two effects (by setting κ, the importance of equity for bank lending, to zero and furthermore making the banking sector frictionless and perfectly competitive so that the deposit rate equals the policy rate), and focus just on liquidity provision. Furthermore, we can focus on a steady state and abstract from time subscripts. What is the optimal interest rate on CBDC in that case?

One can show that under these conditions, the optimal CBDC rate solves the following problem:

$$\max_{1+i^{cbdc}} \quad m + d + cbdc - \Phi(\mathcal{L}(m, d, cbdc)).$$

The F.O.C. for this problem is:

$$\begin{aligned} 0 &= \frac{\partial m}{\partial 1+i^{cbdc}} + \frac{\partial d}{\partial 1+i^{cbdc}} + \frac{\partial cbdc}{\partial 1+i^{cbdc}} - \Phi'(\mathcal{L})\frac{\partial \mathcal{L}}{\partial m}\frac{\partial m}{\partial 1+i^{cbdc}} \\ &- \Phi'(\mathcal{L})\frac{\partial \mathcal{L}}{\partial d}\frac{\partial d}{\partial 1+i^{cbdc}} - \Phi'(\mathcal{L})\frac{\partial \mathcal{L}}{\partial cbdc}\frac{\partial cbdc}{\partial 1+i^{cbdc}}. \end{aligned}$$

We simplify this to:

$$\begin{aligned} 0 &= \frac{\partial m}{\partial 1+i^{cbdc}}\left(1 - \Phi'(\mathcal{L})\frac{\partial \mathcal{L}}{\partial m}\right) + \frac{\partial d}{\partial 1+i^{cbdc}}\left(1 - \Phi'(\mathcal{L})\frac{\partial \mathcal{L}}{\partial d}\right) \\ &+ \frac{\partial cbdc}{\partial 1+i^{cbdc}}\left(1 - \Phi'(\mathcal{L})\frac{\partial \mathcal{L}}{\partial cbdc}\right). \end{aligned}$$

Recalling that the F.O.C.'s of the household problem (equations B.3-B.5 below) take the form:

$$\Phi'(\mathcal{L})\frac{\partial \mathcal{L}}{\partial x} = \frac{1+i^x}{1+i},$$

and using this in the previous expression, we obtain:

$$0 = \frac{\partial m}{\partial 1+i^{cbdc}}(i-i^m) + \frac{\partial d}{\partial 1+i^{cbdc}}(i-i^d) + \frac{\partial cbdc}{\partial 1+i^{cbdc}}(i-i^{cbdc}). \tag{A.16}$$

After a lot of algebra and simplifications we can write the previous expression as:

$$0 = \frac{m}{\mathcal{L}}(i-i^m) + \frac{d}{\mathcal{L}}(i-i^d) + \frac{cbdc}{\mathcal{L}}(i-i^{cbdc}) + \frac{\theta}{\frac{1}{b-1}-\theta}\frac{1+i^{\mathcal{L}}}{1+i^{cbdc}}(i-i^{cbdc}). \tag{A.17}$$

This expression (which is an extension of the Friedman rule to multiple instruments with imperfect substitutability, under satiation, and where the steady state policy rate is considered exogenous and determined, for example, by real factors and ZLB considerations), is very intuitive, and it indicates that the social planner "seeks" to achieve four things simultaneously. It seeks to promote the optimal level of cash usage, deposit usage, CBDC usage, and overall liquidity usage.

While the expression above is useful, if we do some approximations we can obtain even more intuition. We will approximate $x/\mathcal{L}$ with $\omega^x_{\mathcal{L}}$, this is not exact but only differs by terms like $(1+i^x)/(1+i^{\mathcal{L}})$, which will be very close to one. We will also approximate $\theta/(1/(b-1)-\theta)$ by minus one, which is close to being true because $\frac{1}{b-1}$ is around 14 while θ is around 550. Finally, we will also approximate $(1+i^{\mathcal{L}})/(1+i^{cbdc})$ by one. In this case, we get (recall that all three omegas sum to one):

$$\begin{aligned} 0 &= \omega^m_{\mathcal{L}}(i-i^m) + \omega^d_{\mathcal{L}}(i-i^d) + \omega^{cbdc}_{\mathcal{L}}(i-i^{cbdc}) - (i-i^{cbdc}) \\ i^{cbdc} &= \frac{\omega^m_{\mathcal{L}}}{\omega^m_{\mathcal{L}}+\omega^d_{\mathcal{L}}} i^m + \frac{\omega^d_{\mathcal{L}}}{\omega^m_{\mathcal{L}}+\omega^d_{\mathcal{L}}} i^d \end{aligned}$$

So, under this approximation, the result is that, just from the perspective of liquidity provision, the optimal rate on CBDC is set as a weighted average of i^m (which is zero in our model) and i^d, where the weights sum to one and are given by $\omega^x_{\mathcal{L}}/(\omega^m_{\mathcal{L}}+\omega^d_{\mathcal{L}})$. When the policy rate is very low or negative, the weight of cash is higher, so the optimal rate of CBDC tracks the rate on cash (zero) more closely. Conversely, when the policy rate is high, deposits have most of the weight, so the optimal CBDC rate tracks the deposit rate closely.

Appendix B Details on the Full Model

Appendix B.1 The Household's Problem

The Bellman equation for the household's problem is given by:

$$V_t(AH_{t-1}) = \max_{C_t, N_t, M_t, \{D_{j,t}\}_{j=1}^n, CBDC_t, B_t} \left\{ u(C_t) - v(N_t) + \beta \mathbb{E}_t(V_{t+1}(AH_t)) \right\}.$$

We can express C_t as:

$$C_t = \frac{W_t N_t + AH_{t-1} + T_t - B_t - \Phi(\mathcal{L}_t)P_t}{P_t},$$

with this definition we can write the Bellman equation as a function of 4 individual choice variables and n deposit choices (the $D_{j,t}$). The first-order conditions can be written as:

$$
\begin{aligned}
v'(N_t) &= u'(C_t)\frac{W_t}{P_t} && \text{(B.1)}\\
u'(C_t)\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t}\frac{1}{P_t} &= \beta(1+i_t^m)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right)\\
u'(C_t)\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\frac{\partial d_t}{\partial d_{j,t}}\frac{1}{P_t} &= \beta(1+i_{j,t}^d)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right)\\
u'(C_t)\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t}\frac{1}{P_t} &= \beta(1+i_t^{cbdc})\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right)\\
\frac{u'(C_t)}{P_t} &= \beta(1+i_t)\mathbb{E}_t\left(\frac{u'(C_{t+1})}{P_{t+1}}\right). && \text{(B.2)}
\end{aligned}
$$

The first condition is the intratemporal condition for labor supply and the fifth one is the Euler equation. The second, third, and fourth deal with the demand for cash, deposits, and CBDC respectively.

We first aggregate the individual demands for the deposits of each of the n banks into an aggregate deposit demand. If we introduce the fifth F.O.C. into the third, we obtain:

$$
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\frac{\partial d_t}{\partial d_{j,t}} = \frac{1+i_{j,t}^d}{1+i_t}.
$$

The derivative of aggregate deposits w.r.t. an individual deposit is:

$$
\begin{aligned}
\frac{\partial d_t}{\partial d_{j,t}} &= \frac{\varphi^d}{\varphi^d+1}\left(\sum_{j=1}^n \alpha_j^{-\frac{1}{\varphi^d}} d_{j,t}^{\frac{\varphi^d+1}{\varphi^d}}\right)^{-\frac{1}{\varphi^d+1}} \alpha_j^{-\frac{1}{\varphi^d}}\frac{\varphi^d+1}{\varphi^d} d_{j,t}^{\frac{1}{\varphi^d}}\\
&= \left(d_t^{\frac{\varphi^d+1}{\varphi^d}}\right)^{-\frac{1}{\varphi^d+1}} \alpha_j^{-\frac{1}{\varphi^d}} d_{j,t}^{\frac{1}{\varphi^d}} = \alpha_j^{-\frac{1}{\varphi^d}}\left(\frac{d_{j,t}}{d_t}\right)^{\frac{1}{\varphi^d}}.
\end{aligned}
$$

Introducing this into the F.O.C. for deposits we get:

$$
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\alpha_j^{-\frac{1}{\varphi^d}}\left(\frac{d_{j,t}}{d_t}\right)^{\frac{1}{\varphi^d}} = \frac{1+i_{j,t}^d}{1+i_t},
$$

raise this to the power of φ^d+1, multiply by α_j, and then add over banks:

$$
\begin{aligned}
\left(\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t}\right)^{\varphi^d+1}\alpha_j^{-\frac{\varphi^d+1}{\varphi^d}}\left(\frac{d_{j,t}}{d_t}\right)^{\frac{\varphi^d+1}{\varphi^d}} &= \frac{(1+i_{j,t}^d)^{\varphi^d+1}}{(1+i_t)^{\varphi^d+1}}\\
\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} &= \frac{1+i_t^d}{1+i_t},
\end{aligned}
$$

where we have defined:

$$
1+i_t^d = \left(\sum_{j=1}^n \alpha_j(1+i_{j,t}^d)^{\varphi^d+1}\right)^{\frac{1}{\varphi^d+1}}.
$$

Using the equation $\Phi'(\mathcal{L}_t)(\partial\mathcal{L}_t/\partial d_t) = (1+i_t^d)/(1+i_t)$, we can turn the F.O.C. for individual deposits into:

$$d_{j,t} = \alpha_j \left(\frac{1+i^d_{j,t}}{1+i^d_t} \right)^{\varphi^d} d_t.$$

Once we have "aggregated up" deposits, we can turn to the decision between the three liquid savings instruments, where we have the following three F.O.C.s:

$$\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial m_t} = \frac{1+i^m_t}{1+i_t} \tag{B.3}$$

$$\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial d_t} = \frac{1+i^d_t}{1+i_t} \tag{B.4}$$

$$\Phi'(\mathcal{L}_t)\frac{\partial \mathcal{L}_t}{\partial cbdc_t} = \frac{1+i^{cbdc}_t}{1+i_t}. \tag{B.5}$$

The derivative of liquidity w.r.t. real money balances is:

$$\frac{\partial \mathcal{L}_t}{\partial m_t} = \frac{\theta}{\theta+1}\left(\mathcal{L}_t^{\frac{\theta+1}{\theta}}\right)^{\frac{\theta}{\theta+1}-1} \gamma_m^{-\frac{1}{\theta}} \frac{\theta+1}{\theta} m_t^{\frac{1}{\theta}} = \mathcal{L}_t^{-\frac{1}{\theta}} \gamma_m^{-\frac{1}{\theta}} m_t^{\frac{1}{\theta}}.$$

Similar expressions are available for $\partial \mathcal{L}_t / \partial d_t$ and $\partial \mathcal{L}_t / \partial cbdc_t$. We can write these demands as

$$\begin{aligned}
\Phi'(\mathcal{L}_t)^{\theta+1} \mathcal{L}_t^{-\frac{\theta+1}{\theta}} \gamma_m^{-\frac{1}{\theta}} m_t^{\frac{\theta+1}{\theta}} &= \gamma_m \frac{(1+i^m_t)^{\theta+1}}{(1+i_t)^{\theta+1}} \\
\Phi'(\mathcal{L}_t)^{\theta+1} \mathcal{L}_t^{-\frac{\theta+1}{\theta}} \gamma_d^{-\frac{1}{\theta}} d_t^{\frac{\theta+1}{\theta}} &= \gamma_d \frac{(1+i^d_t)^{\theta+1}}{(1+i_t)^{\theta+1}} \\
\Phi'(\mathcal{L}_t)^{\theta+1} \mathcal{L}_t^{-\frac{\theta+1}{\theta}} \gamma_{cbdc}^{-\frac{1}{\theta}} cbdc_t^{\frac{\theta+1}{\theta}} &= \gamma_{cbdc} \frac{(1+i^{cbdc}_t)^{\theta+1}}{(1+i_t)^{\theta+1}},
\end{aligned}$$

by adding these three we get:

$$\Phi'(\mathcal{L}_t)^{\theta+1} \mathcal{L}_t^{-\frac{\theta+1}{\theta}} \mathcal{L}_t^{\frac{\theta+1}{\theta}} = \frac{(1+i^{\mathcal{L}}_t)^{\theta+1}}{(1+i_t)^{\theta+1}},$$

where the aggregate interest rate for liquidity takes the form:

$$1+i^{\mathcal{L}}_t \equiv \left(\gamma_m (1+i^m_t)^{\theta+1} + \gamma_d (1+i^d_t)^{\theta+1} + \gamma_{cbdc}(1+i^{cbdc}_t)^{\theta+1}\right)^{\frac{1}{\theta+1}}. \tag{B.6}$$

This finally allows us to write a simple demand equation for overall liquidity:

$$\frac{1+i^{\mathcal{L}}_t}{1+i_t} = \Phi'(\mathcal{L}_t). \tag{B.7}$$

And we can write the demand for each instrument as:

$$m_t = \gamma_m \left(\frac{1+i^m_t}{1+i^{\mathcal{L}}_t}\right)^{\theta} \mathcal{L}_t \tag{B.8}$$

$$d_t = \gamma_d \left(\frac{1+i^d_t}{1+i^{\mathcal{L}}_t}\right)^{\theta} \mathcal{L}_t \tag{B.9}$$

$$cbdc_t = \gamma_{cbdc} \left(\frac{1 + i_t^{cbdc}}{1 + i_t^{\mathcal{L}}} \right)^{\theta} \mathcal{L}_t. \tag{B.10}$$

Appendix B.2 Alternative Setup: Liquidity in Utility

In the baseline model, liquid instruments are demanded by the household because of the non-linear cost function $\Phi(\mathcal{L}_t)$ in the budget constraint. This leads to the set of tractable holding schedules in (3.5)-(3.7). This appendix provides an alternative setup where we introduce liquidity into the utility function to achieve the same holding schedules. Assuming all banks are symmetric, we focus only on the aggregate deposits.

Assume the household has the following utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (U(C_t, m_t, d_t, cbdc_t) - v(N_t)),$$

while keeping the budget constraint standard:

$$P_t C_t + B_t + M_t + D_t + CBDC_t = W_t N_t + AH_{t-1} + T_t,$$

where AH_{t-1} is the same as in the main text. The liquidity instruments now have a one-for-one cost in the budget constraint, but they enter the utility function.

The first-order conditions are

$$\begin{aligned}
v'(N_t) &= U_C(C_t, m_t, d_t, cbdc_t) \frac{W_t}{P_t} \\
\frac{U_C(C_t, m_t, d_t, cbdc_t)}{P_t} &= \frac{U_m(C_t, m_t, d_t, cbdc_t)}{P_t} + \beta(1 + i_t^m)\mathbb{E}_t \left(\frac{U_C(C_{t+1}, m_{t+1}, d_{t+1}, cbdc_{t+1}))}{P_{t+1}} \right) \\
\frac{U_C(C_t, m_t, d_t, cbdc_t)}{P_t} &= \frac{U_d(C_t, m_t, d_t, cbdc_t)}{P_t} + \beta(1 + i_t^d)\mathbb{E}_t \left(\frac{U_C(C_{t+1}, m_{t+1}, d_{t+1}, cbdc_{t+1}))}{P_{t+1}} \right) \\
\frac{U_C(C_t, m_t, d_t, cbdc_t)}{P_t} &= \frac{U_{cbdc}(C_t, m_t, d_t, cbdc_t)}{P_t} + \beta(1 + i_t^{cbdc})\mathbb{E}_t \left(\frac{U_C(C_{t+1}, m_{t+1}, d_{t+1}, cbdc_{t+1}))}{P_{t+1}} \right) \\
\frac{U_C(C_t, m_t, d_t, cbdc_t)}{P_t} &= \beta(1 + i_t)\mathbb{E}_t \left(\frac{U_C(C_{t+1}, m_{t+1}, d_{t+1}, cbdc_{t+1})}{P_{t+1}} \right).
\end{aligned}$$

Introducing the last equation (Euler equation) into the second to fourth ones and simplifying, we obtain:

$$\frac{i_t - i_t^m}{1 + i_t} = \frac{U_m(C_t, m_t, d_t, cbdc_t)}{U_C(C_t, m_t, d_t, cbdc_t)} \tag{B.11}$$

$$\frac{i_t - i_t^d}{1 + i_t} = \frac{U_d(C_t, m_t, d_t, cbdc_t)}{U_C(C_t, m_t, d_t, cbdc_t)} \tag{B.12}$$

$$\frac{i_t - i_t^{cbdc}}{1 + i_t} = \frac{U_{cbdc}(C_t, m_t, d_t, cbdc_t)}{U_C(C_t, m_t, d_t, cbdc_t)}. \tag{B.13}$$

Let's assume a non-separable utility function similar to Greenwood et al. (1988), with total utility taking the following form:

$$U(C_t, m_t, d_t, cbdc_t) = \frac{(C_t + \xi(m_t, d_t, cbdc_t))^{1-\sigma} - 1}{1 - \sigma},$$

then the marginal utilities with respect to consumption and the liquid instruments take the following forms:

$$\begin{aligned} U_x(C_t, m_t, d_t, cbdc_t) &= (C_t + \xi(m_t, d_t, cbdc_t))^{-\sigma} \frac{\partial \xi}{\partial x} \quad \text{for } x = m, d, cbdc \\ U_C(C_t, m_t, d_t, cbdc_t) &= (C_t + \xi(m_t, d_t, cbdc_t))^{-\sigma}. \end{aligned}$$

Hence, (B.11)-(B.13) become

$$\frac{i_t - i_t^m}{1 + i_t} = \frac{\partial \xi_t}{\partial m_t} \tag{B.14}$$

$$\frac{i_t - i_t^d}{1 + i_t} = \frac{\partial \xi_t}{\partial d_t} \tag{B.15}$$

$$\frac{i_t - i_t^{cbdc}}{1 + i_t} = \frac{\partial \xi_t}{\partial cbdc_t}. \tag{B.16}$$

These equations are convenient, because they do not contain wealth effects in the demand for cash, deposits, and CBDC. This result comes from the GHH-style non-separable utility function.

Next, we assume the ξ function takes the following form:

$$\xi(m_t, d_t, cbdc_t) = m_t + d_t + cbdc_t - \Phi(\mathcal{L}_t(m_t, d_t, cbdc_t)),$$

Taking its derivatives with respect to money, aggregate deposits, and CBDC, we get

$$\begin{aligned} \frac{\partial \xi_t}{\partial m_t} &= 1 - \Phi'(\mathcal{L}_t) \frac{\partial \mathcal{L}_t}{\partial m_t} \\ \frac{\partial \xi_t}{\partial d_t} &= 1 - \Phi'(\mathcal{L}_t) \frac{\partial \mathcal{L}_t}{\partial d_t} \\ \frac{\partial \xi_t}{\partial cbdc_t} &= 1 - \Phi'(\mathcal{L}_t) \frac{\partial \mathcal{L}_t}{\partial cbdc_t}, \end{aligned}$$

turning (B.14)-(B.16) into

$$\begin{aligned} \Phi'(\mathcal{L}_t) \frac{\partial \mathcal{L}_t}{\partial m_t} &= \frac{1 + i_t^m}{1 + i_t} \\ \Phi'(\mathcal{L}_t) \frac{\partial \mathcal{L}_t}{\partial d_t} &= \frac{1 + i_t^d}{1 + i_t} \\ \Phi'(\mathcal{L}_t) \frac{\partial \mathcal{L}_t}{\partial cbdc_t} &= \frac{1 + i_t^{cbdc}}{1 + i_t}. \end{aligned}$$

These equations are identical to equations (B.3)-(B.5), which then lead to the holding schedules (3.5)-(3.7).

Appendix B.3 The Intermediate Good Firm's Problem

The Bellman equation of the intermediate good firm is:

$$V_t(\{K_{j,t}^P\}_{j=1}^n, K_t^{NP}) = \max_{N_t, \{K_{j,t+1}^P\}_{j=1}^n, K_{t+1}^{NP}} \left\{ \Pi_t^m + \mathbb{E}_t(\Lambda_{t,t+1} V_{t+1}(\{K_{j,t+1}^P\}_{j=1}^n, K_{t+1}^{NP}) \right\},$$

where $\Lambda_{t,t+1}$ is the sdf that the household uses to discount nominal cash flows between $t+1$ and t. The derivatives of K_t w.r.t. non-pledgeable and the different components of pledgeable capital are:

$$\begin{aligned}
\frac{\partial K_t}{\partial K_t^{NP}} &= (1-\psi)^{\frac{1}{\theta^k}} \left(\frac{K_t}{K_t^{NP}}\right)^{\frac{1}{\theta^k}} \\
\frac{\partial K_t}{\partial K_{j,t}^P} &= \frac{\partial K_t}{\partial K_t^P}\frac{\partial K_t^P}{\partial K_{j,t}^P} = \psi^{\frac{1}{\theta^k}} \left(\frac{K_t}{K_t^P}\right)^{\frac{1}{\theta^k}} (\alpha_j^l)^{\frac{1}{\varphi^l}} \left(\frac{K_t^P}{K_{j,t}^P}\right)^{\frac{1}{\varphi^l}}.
\end{aligned}$$

The F.O.C.'s w.r.t. labor, non-pledgeable, and all the individual types of pledgeable capital are then:

$$\begin{aligned}
0 &= (1-\alpha)P_t^m \frac{Y_t^m}{N_t} - W_t \\
0 &= \mathbb{E}_t\left(\Lambda_{t,t+1}\frac{\partial V_{t+1}(\{K_{j,t+1}^P\}_{j=1}^n, K_{t+1}^{NP})}{\partial K_{t+1}^{NP}}\right) \\
0 &= \mathbb{E}_t\left(\Lambda_{t,t+1}\frac{\partial V_{t+1}(\{K_{j,t+1}^P\}_{j=1}^n, K_{t+1}^{NP})}{\partial K_{j,t+1}^P}\right).
\end{aligned}$$

The Benveniste-Scheinkman conditions are:

$$\begin{aligned}
\frac{\partial V_t(\{K_{j,t}^P\}_{j=1}^n, K_t^{NP})}{\partial K_t^{NP}} &= \alpha(1-\psi)^{\frac{1}{\theta^k}} P_t^m \frac{Y_t^m}{K_t}\left(\frac{K_t}{K_t^{NP}}\right)^{\frac{1}{\theta^k}} + (1-\delta)Q_t - Q_{t-1}(1+i_{t-1}+\varrho) \\
\frac{\partial V_t(\{K_{j,t}^P\}_{j=1}^n, K_t^{NP})}{\partial K_{j,t}^P} &= \alpha\psi^{\frac{1}{\theta^k}} P_t^m \frac{Y_t^m}{K_t}\left(\frac{K_t}{K_t^P}\right)^{\frac{1}{\theta^k}} (\alpha_j^l)^{\frac{1}{\varphi^l}} \left(\frac{K_t^P}{K_{j,t}^P}\right)^{\frac{1}{\varphi^l}} + (1-\delta)Q_t - Q_{t-1}(1+i_{j,t-1}^l).
\end{aligned}$$

Moving these forward one period and introducing them in the capital F.O.C.s we get:

$$0 = \mathbb{E}_t\left(\Lambda_{t,t+1}\left(\alpha\psi^{\frac{1}{\theta^k}} P_{t+1}^m \frac{Y_{t+1}^m}{K_{t+1}}\left(\frac{K_{t+1}}{K_{t+1}^P}\right)^{\frac{1}{\theta^k}} (\alpha_j^l)^{\frac{1}{\varphi^l}} \left(\frac{K_{t+1}^P}{K_{j,t+1}^P}\right)^{\frac{1}{\varphi^l}} + (1-\delta)Q_{t+1} - Q_t(1+i_{j,t}^l)\right)\right).$$

Using the fact that $\Lambda_{t,t+1} = \beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_t}{P_{t+1}}$, the Euler equation, and denoting the intermediate variable $\Theta_t \equiv \mathbb{E}_t\left(\alpha\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P_{t+1}^m}{P_{t+1}}\frac{Y_{t+1}^m}{K_{t+1}}\right)$ we obtain:

$$\frac{Q_t}{P_t}\frac{1+i_{j,t}^l}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right) = \Theta_t\psi^{\frac{1}{\theta^k}}\left(\frac{K_{t+1}}{K_{t+1}^P}\right)^{\frac{1}{\theta^k}} (\alpha_j^l)^{\frac{1}{\varphi^l}} \left(\frac{K_{t+1}^P}{K_{j,t+1}^P}\right)^{\frac{1}{\varphi^l}}.$$

We manipulate the second equation (of which there are a total of n versions, one for each bank), raising it to the power of $1-\varphi^l$, multiplying by α_j^l, and then adding over all the n equations, to obtain:

$$\sum_{j=1}^n \alpha_j^l (z_{j,t}^P)^{1-\varphi^l} = \left(\Theta_t\psi^{\frac{1}{\theta^k}}\left(\frac{K_{t+1}}{K_{t+1}^P}\right)^{\frac{1}{\theta^k}}\right)^{1-\varphi^l} \sum_{j=1}^n (\alpha_j^l)^{\frac{1}{\varphi^l}} \left(\frac{K_{j,t+1}^P}{K_{t+1}^P}\right)^{\frac{\varphi^l-1}{\varphi^l}},$$

where,

$$z_{j,t}^P \equiv \frac{Q_t}{P_t}\frac{1+i_{j,t}^l}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right).$$

Defining:

$$z_t^P \equiv \left(\sum_{j=1}^n \alpha_j^l (z_{j,t}^P)^{1-\varphi^l}\right)^{\frac{1}{1-\varphi^l}},$$

we can rewrite the previous expression as:

$$z_t^P = \Theta_t \psi^{\frac{1}{\theta^k}} \left(\frac{K_{t+1}}{K_{t+1}^P}\right)^{\frac{1}{\theta^k}}.$$

We can also write demand for the individual pledgeable capital of bank j as:

$$K_{j,t+1}^P = \alpha_j^l \left(\frac{z_{j,t}^P}{z_t^P}\right)^{-\varphi^l} K_{t+1}^P.$$

Defining:

$$z_t^{NP} \equiv \frac{Q_t}{P_t}\frac{1+i_t+\varrho}{1+i_t} - (1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right),$$

we then have two aggregate conditions for K_t^{NP} and K_t^P that we can rewrite as:

$$\begin{aligned}
\Theta_t(1-\psi)^{\frac{1}{\theta^k}} K_{t+1}^{\frac{1}{\theta^k}} (K_{t+1}^{NP})^{-\frac{1}{\theta^k}} &= z_t^{NP} \\
\Theta_t \psi^{\frac{1}{\theta^k}} K_{t+1}^{\frac{1}{\theta^k}} (K_{t+1}^{P})^{-\frac{1}{\theta^k}} &= z_t^{P}.
\end{aligned}$$

Simplify to obtain:

$$\left(\Theta_t K_{t+1}^{\frac{1}{\theta^k}}\right)^{1-\theta^k} K_{t+1}^{\frac{\theta^k-1}{\theta^k}} = z_t^{1-\theta^k},$$

where:

$$z_t \equiv \left(\psi (z_t^P)^{1-\theta^k} + (1-\psi)(z_t^{NP})^{1-\theta^k}\right)^{\frac{1}{1-\theta^k}}.$$

The previous equation for determining aggregate K_t as a function of z_t can then be simplified to:

$$\Theta_t = z_t.$$

With this, the F.O.C.'s for pledgeable and non-pledgeable capital can also be expressed as:

$$K_{t+1}^{NP} = (1-\psi)\left(\frac{z_t^{NP}}{z_t}\right)^{-\theta^k} K_{t+1}$$

$$K_{t+1}^P = \psi \left(\frac{z_t^P}{z_t} \right)^{-\theta^k} K_{t+1}.$$

Appendix B.4 The Capital Producer

We assume that even though non-pledgeable and pledgeable capital are financed differently by intermediate good firms, they are produced by the same representative capital producer that treats them indistinguishably, so they have the same price of capital Q_t and there is a single investment adjustment cost. It would be straightforward to augment the model to have two different prices of capital. Denote:

$$K_t^S = K_t^{NP} + \sum_{j=1}^{n} K_{j,t}^P.$$

The representative capital producer sells $Q_t K_{t+1}^S$ dollars worth of new capital, buys $(1-\delta) Q_t K_t^S$ dollars worth of used capital, and additionally pays I_t dollars in order to increase capital from K_t^S to K_{t+1}^S. New capital K_{t+1}^S is obtained from K_t^S and I_t as follows:

$$K_{t+1}^S = (1-\delta) K_t^S + I_t \left(1 - \Xi \left(\frac{I_t}{I_{t-1}} \right) \right).$$

With these elements, the nominal period-t profits of the capital good producer are:

$$\Pi_t^K = Q_t K_{t+1}^S - (1-\delta) Q_t K_t^S - P_t I_t,$$

which, using the previous equation for K_{t+1}^S, can be expressed as:

$$\Pi_t^K = Q_t I_t \left(1 - \Xi \left(\frac{I_t}{I_{t-1}} \right) \right) - P_t I_t,$$

where $\Xi(\cdot)$ captures investment adjustment costs. The problem of the capital producer in period t is:

$$\max_{I_t} \ \mathbb{E}_t \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} \left[Q_{t+\tau} I_{t+\tau} \left(1 - \Xi \left(\frac{I_{t+\tau}}{I_{t+\tau-1}} \right) \right) - P_{t+\tau} I_{t+\tau} \right],$$

where $\Lambda_{t,t+\tau}$ is the household's nominal stochastic discount factor for discounting nominal flows from $t+\tau$ back to t. The F.O.C. is:

$$1 = \frac{Q_t}{P_t} \left[1 - \Xi \left(\frac{I_t}{I_{t-1}} \right) - \Xi' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \mathbb{E}_t \beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{Q_{t+1}}{P_{t+1}} \Xi' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2.$$

The $\Xi(\cdot)$ function satisfies $\Xi(1) = \Xi'(1) = 0$ and $\Xi''(1) \geq 0$.

Appendix B.5 The Bank's Problem

Appendix B.5.1 Steady-State Bank Return on Equity

Here we show that in any steady state of our model, pre-CBDC or post-CBDC for any CBDC remuneration schedule, bank return on equity is the same. Start by using equation (3.15) in steady state:

$$\frac{F_j}{P} = \frac{F_j}{P}(1-\varsigma) + \omega \frac{X_j}{P}.$$

Simplify this expression to obtain:

$$\frac{\varsigma}{\omega} = \frac{X_j}{F_j}. \tag{B.17}$$

Additionally, since we assume zero steady-state inflation, from the definition of X_j we have:

$$\frac{X_j}{F_j} = i + (i_j^l - \mu^l - i)\frac{L_j}{F_j} + (i - \mu^d - i_j^d)\frac{D_j}{F_j} - \Psi\left(\frac{L_j}{F_j}\right). \tag{B.18}$$

From the definition of end-of-period resources we know:

$$S_j = (1 + i_j^l - \mu^l)L_j + (1+i)H_j - (1 + i_j^d + \mu^d)D_j - \varsigma F_j - \Psi\left(\frac{L_j}{F_j}\right)F_j.$$

Introducing the bank balance sheet constraint in the previous expression, dividing by F_j, and simplifying, we obtain:

$$\frac{S_j}{F_j} = 1 + i - \varsigma + (i_j^l - \mu^l - i)\frac{L_j}{F_j} + (i - \mu^d - i_j^d)\frac{D_j}{F_j} - \Psi\left(\frac{L_j}{F_j}\right). \tag{B.19}$$

Using (B.18) in (B.19), we get:

$$\frac{S_j}{F_j} = 1 - \varsigma + \frac{X_j}{F_j}.$$

Further using (B.17), we finally obtain

$$\frac{S_j}{F_j} - 1 = \varsigma\left(\frac{1}{\omega} - 1\right) \tag{B.20}$$

The object on the left hand side is the net return on equity of bank j: end of period resources divided by initial equity minus one. Since the right hand side does not depend on whether CBDC is present or not (or on the CBDC remuneration schedule), we have shown that CBDC introduction leaves bank return on equity unchanged from the pre-CBDC steady state to the post-CBDC one.

Appendix B.5.2 Separation

Recall that the bank's problem is given by:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s+1} DIV_{j,t+s+1}.$$

As discussed in the main text, banks do not independently optimize their dividend distribution but instead take as given that a fraction $(1-\omega)$ of "profits" $X_{j,t+1}$ are distributed as dividends. The Bellman equation for the bank's problem is:

$$V(F_{j,t},\Omega_t) = \max_{i^d_{j,t},D_{j,t},i^l_{j,t},L_{j,t}} \mathbb{E}\left\{\Lambda DIV_{j,t+1} + \Lambda V(F_{j,t+1},\Omega_{t+1})\right\},$$

where Ω_t denotes the aggregate state variables that influence the value of being a bank in period t and we have used Λ as short hand notation for $\Lambda_{t,t+1}$. The maximization problem is subject to the deposit supply schedule, loan demand schedule, as well as:

$$\begin{aligned} DIV_{j,t+1} &= (1-\omega)X_{j,t+1} \\ F_{j,t+1} &= F_{j,t}(1-\varsigma)(1+\pi_{t+1}) + \omega X_{j,t+1} \\ X_{j,t+1} &= i_t F_{j,t} + (i^l_{j,t} - \mu^l - i_t)L_{j,t} + (i_t - \mu^d - i^d_{j,t})D_{j,t} \\ &- \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t} - F_{j,t}(1-\varsigma)\pi_{t+1}. \end{aligned}$$

The F.O.C. w.r.t. i^d_j yields the following:

$$0 = \mathbb{E}\left\{\Lambda(1-\omega)\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}} + \Lambda\frac{\partial V(F_{j,t+1},\Omega_{t+1})}{\partial F_{j,t+1}}\omega\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}}\right\}.$$

Since $\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}}$ is deterministic (known in period t), it can exit the expectation operator and the optimality condition becomes $\frac{\partial X_{j,t+1}}{\partial i^d_{j,t}} = 0$, which is equivalent to maximizing $(i_t - \mu^d - i^d_{j,t})D_{j,t}$ w.r.t. $i^d_{j,t}$ subject to the deposit supply schedule $D_{j,t}(i^d_{j,t})$.

Similarly, the F.O.C. w.r.t. i^l_j yields the following:

$$0 = \mathbb{E}\left\{\Lambda(1-\omega)\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}} + \Lambda\frac{\partial V(F_{j,t+1},\Omega_{t+1})}{\partial F_{j,t+1}}\omega\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}}\right\}.$$

Since $\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}}$ is also deterministic, it can exit the expectation operator as well, and the optimality condition becomes $\frac{\partial X_{j,t+1}}{\partial i^l_{j,t}} = 0$, which is equivalent to maximizing

$$(i^l_{j,t} - \mu^l - i_t)L_{j,t} - \Psi\left(\frac{L_{j,t}}{F_{j,t}}\right)F_{j,t}$$

w.r.t. $i^l_{j,t}$ subject to the loan demand schedule $L_{j,t}(i^l_{j,t})$.

The reason the deposit and loan problems can be neatly separated, is because banks can always use their reserves $H_{j,t}$ to borrow or lend any excess funds to the central bank, so they always optimize their loan and deposit franchises separately. If there was a constraint like $H_{j,t} \geq 0$, then there are circumstances under which the deposit and loan franchises interact and the maximization problem cannot be neatly separated into the two subproblems.

Appendix B.5.3 Deposits

A bank that maximizes $(i_t - i^d_{j,t} - \mu^d)D_{j,t}$ has the following F.O.C.:

$$0 = -D_{j,t} + ((1+i_t-\mu^d)-(1+i^d_{j,t}))\frac{\partial D_{j,t}}{\partial d_{j,t}}\frac{\partial d_{j,t}}{\partial(1+i^d_{j,t})}.$$

Denote with $\epsilon^d_{j,t}$ the endogenous elasticity of $d_{j,t}$ w.r.t. $(1+i^d_{j,t})$:

$$\epsilon^d_{j,t} \equiv \frac{\partial d_{j,t}}{\partial(1+i^d_{j,t})}\frac{1+i^d_{j,t}}{d_{j,t}}.$$

Then we can write the previous F.O.C. as:

$$\begin{aligned} 1 &= \epsilon^d_{j,t}((1+i_t-\mu^d)-(1+i^d_{j,t}))\frac{1}{1+i^d_{j,t}} \\ 1+i^d_{j,t} &= \frac{\epsilon^d_{j,t}}{\epsilon^d_{j,t}+1}(1+i_t-\mu^d). \end{aligned} \tag{B.21}$$

Now, let's obtain $\epsilon^d_{j,t}$. This is not trivial because both $1+i^d_t$ and d_t depend on $1+i^d_{j,t}$. Let's compute the elasticity of the aggregate deposit rate w.r.t. one individual deposit rate:

$$\begin{aligned} \frac{\partial(1+i^d_t)}{\partial(1+i^d_{j,t})} &= \frac{1}{\varphi^d+1}\left(\sum_{j=1}^n \alpha_j(1+i^d_{j,t})^{\varphi^d+1}\right)^{-\frac{\varphi^d}{\varphi^d+1}}\alpha_j(\varphi^d+1)(1+i^d_{j,t})^{\varphi^d} \\ \frac{\partial(1+i^d_t)}{\partial(1+i^d_{j,t})}\frac{1+i^d_{j,t}}{1+i^d_t} &= \alpha_j\left(\frac{1+i^d_{j,t}}{1+i^d_t}\right)^{\varphi^d+1} = \frac{(1+i^d_{j,t})d_{j,t}}{(1+i^d_t)d_t} \equiv \omega^{d_j}_{d,t}, \end{aligned} \tag{B.22}$$

where $\omega^{d_j}_{d,t}$ is the share of gross interest spending on deposits of bank j at time t. Now let's compute the elasticity of d_t w.r.t. $(1+i^d_t)$:

$$\frac{\partial \ln d_t}{\partial \ln(1+i^d_t)} = \theta - \theta\frac{\partial \ln(1+i^{\mathcal{L}}_t)}{\partial \ln(1+i^d_t)} + \frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i^{\mathcal{L}}_t)}\frac{\partial \ln(1+i^{\mathcal{L}}_t)}{\partial \ln(1+i^d_t)}$$

The elasticity of $1+i^{\mathcal{L}}_t$ w.r.t. $1+i^d_t$ is:

$$\begin{aligned} \frac{\partial(1+i^{\mathcal{L}}_t)}{\partial(1+i^d_t)} &= \left(1+i^{\mathcal{L}}_t\right)^{-\theta}\gamma_d(1+i^d_t)^{\theta} = \frac{d_t}{\mathcal{L}_t} \\ \frac{\partial(1+i^{\mathcal{L}}_t)}{\partial(1+i^d_t)}\frac{1+i^d_t}{1+i^{\mathcal{L}}_t} &= \gamma_d\left(\frac{1+i^d_t}{1+i^{\mathcal{L}}_t}\right)^{\theta+1} = \frac{(1+i^d_t)d_t}{(1+i^{\mathcal{L}}_t)\mathcal{L}_t} \equiv \omega^d_{\mathcal{L},t}. \end{aligned}$$

With all of these things we can write:

$$\begin{aligned} \ln d_{j,t} &= \ln\alpha_j + \varphi^d\ln(1+i^d_{j,t}) - \varphi^d\ln(1+i^d_t) + \ln d_t \\ \frac{\partial \ln d_{j,t}}{\partial \ln(1+i^d_{j,t})} &= \varphi^d - \varphi^d\frac{\partial \ln(1+i^d_t)}{\partial \ln(1+i^d_{j,t})} + \frac{\partial \ln d_t}{\partial \ln(1+i^d_t)}\frac{\partial \ln(1+i^d_t)}{\partial \ln(1+i^d_{j,t})} \\ \epsilon^d_{j,t} &= (1-\omega^{d_j}_{d,t})\varphi^d + \omega^{d_j}_{d,t}\left[(1-\omega^d_{\mathcal{L},t})\theta + \omega^d_{\mathcal{L},t}\frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i^{\mathcal{L}}_t)}\right]. \end{aligned}$$

If all banks are identical they all pay the same deposit rate ($i^d_{j,t} = i^d_t$), all face the same elasticity ϵ^d_t, and they all obtain one n-th of total deposits (i.e. $\omega^{d_j}_{d,t} = 1/n$), and the expression becomes:

$$\epsilon^d_t = \frac{n-1}{n}\varphi^d + \frac{\theta}{n} + \frac{1}{n}\omega^d_{\mathcal{L},t}\left(\frac{\partial \ln \mathcal{L}_t}{\partial \ln(1+i^{\mathcal{L}}_t)} - \theta\right). \tag{B.23}$$

Appendix B.5.4 Loans

The F.O.C. of the loan sub-problem (w.r.t. $i^l_{j,t}$) is:

$$1 + i^l_{j,t} = \frac{\epsilon^l_{j,t}}{\epsilon^l_{j,t} - 1}\left[1 + i_t + \mu^l + \Psi'\left(\frac{L_{j,t}}{F_{j,t}}\right)\right]. \tag{B.24}$$

where $\epsilon^l_{j,t}$ denotes the (negative of the) elasticity of $l_{j,t}$ w.r.t. $(1 + i^l_{j,t})$:

$$\epsilon^l_{j,t} \equiv -\frac{\partial l_{j,t}}{\partial(1+i^l_{j,t})}\frac{1+i^l_{j,t}}{l_{j,t}}.$$

We obtain:

$$\begin{aligned}
\ln l_{j,t} &= \ln \alpha^l_j - \varphi^l \ln z^P_{j,t} + \varphi^l \ln z^P_t + \ln l_t \\
\frac{\partial \ln l_{j,t}}{\partial \ln(1+i^l_{j,t})} &= \left[-\varphi^l + \varphi^l \frac{\partial z^P_t}{\partial z^P_{j,t}} + \frac{\partial \ln l_t}{\partial \ln z^P_{j,t}}\right]\frac{\partial \ln z^P_{j,t}}{\partial \ln(1+i^l_{j,t})} \\
\epsilon^l_{j,t} &= \left[\varphi^l(1-\omega^{l_j}_{l,t}) + \omega^{l_j}_{l,t}\left(\theta^k(1-\omega^{K_P}_{K,t}) + \frac{\omega^{K_P}_{K,t}}{1-\alpha}\right)\right]\frac{Q_t}{P_t}\frac{1+i^l_{j,t}}{1+i_t}\frac{1}{z^P_{j,t}}.
\end{aligned}$$

If all banks are identical, they all charge the same loan rate ($i^l_{j,t} = i^l_t$), face the same elasticity ϵ^l_t, and obtain one n-th of total loans (i.e. $\omega^{l_j}_{l,t} = 1/n$), and the expression becomes:

$$\epsilon^l_t = \left[\frac{n-1}{n}\varphi^l + \frac{1}{n}\left(\theta^k(1-\omega^{K_P}_{K,t}) + \frac{\omega^{K_P}_{K,t}}{1-\alpha}\right)\right]\frac{Q_t}{P_t}\frac{1+i^l_t}{1+i_t}\frac{1}{z^P_t}. \tag{B.25}$$

Appendix B.6 The Retailer's Problem

Recall that the retailer's problem is:

$$\max_{P^*_t} \ \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{u'(C_t)}\frac{P_t}{P_{t+r}}\left[P^*_t - P^m_{t+r}\right] Y_{t+r|t}.$$

Notice that $Y_{t+r|t}$, the amount sold in period $t+r$ by a firm that last reset its price in period t, is defined as:

$$Y_{t+r|t} \equiv \left(\frac{P^*_t}{P_{t+r}}\right)^{-\phi} Y_{t+r}.$$

Hence, its derivative with respect to the optimal reset price is given by:

$$\frac{\partial Y_{t+r|t}}{\partial P_t^*} = -\phi \frac{Y_{t+r|t}}{P_t^*}.$$

The F.O.C. w.r.t. the optimal reset price is then given by:

$$0 = \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{u'(C_t)} \frac{P_t}{P_{t+r}} \left[Y_{t+r|t} - \phi(P_t^* - P_{t+r}^m) \frac{Y_{t+r|t}}{P_t^*} \right]$$

Define

$$\begin{aligned} \Gamma_t^1 &\equiv \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{P_{t+r}} \left(\frac{P_t}{P_{t+r}} \right)^{-\phi} Y_{t+r} P_{t+r}^m \\ \Gamma_t^2 &\equiv \mathbb{E}_t \sum_{r=0}^{\infty} \gamma^r \beta^r \frac{u'(C_{t+r})}{P_{t+r}} \left(\frac{P_t}{P_{t+r}} \right)^{-\phi} Y_{t+r} P_t^*. \end{aligned}$$

With this notation we can write the F.O.C. as:

$$\phi \Gamma_t^1 = (\phi - 1) \Gamma_t^2. \tag{B.26}$$

We can also characterize Γ_t^1 recursively as:

$$\Gamma_t^1 = u'(C_t) \frac{P_t^m}{P_t} Y_t + \gamma \beta \mathbb{E}_t \left(\frac{P_t}{P_{t+1}} \right)^{-\phi} \Gamma_{t+1}^1. \tag{B.27}$$

Similarly, for Γ_t^2 we have:

$$\Gamma_t^2 = u'(C_t) \frac{P_t^*}{P_t} Y_t + \gamma \beta \mathbb{E}_t \frac{P_t^*}{P_{t+1}^*} \left(\frac{P_t}{P_{t+1}} \right)^{-\phi} \Gamma_{t+1}^2. \tag{B.28}$$

From the definition of the price index we can easily derive an equation for its evolution in terms of the real optimal reset price:

$$1 = (1 - \gamma) \left(\frac{P_t^*}{P_t} \right)^{1-\phi} + \gamma \left(\frac{P_{t-1}}{P_t} \right)^{1-\phi}. \tag{B.29}$$

Additionally, the aggregate demand for intermediate inputs is the integral over all retail firms:

$$Y_t^m = \int_0^1 Y_t(s) ds = \int_0^1 \left(\frac{P_t(s)}{P_t} \right)^{-\phi} Y_t ds = Y_t v_t^p, \tag{B.30}$$

where v_t^p is an index of price dispersion that evolves as follows:

$$v_t^p = \int_0^1 \left(\frac{P_t(s)}{P_t} \right)^{-\phi} ds = \gamma \left(\frac{P_{t-1}}{P_t} \right)^{-\phi} v_{t-1}^p + (1 - \gamma) \left(\frac{P_t^*}{P_t} \right)^{-\phi}. \tag{B.31}$$

Equations (B.26)-(B.31) are the ones given in the text as describing the optimal behavior of retail firms.

Appendix B.7 Equilibrium Equations

We assume the following functional forms for $v(\cdot)$, $u(\cdot)$, $\Psi(\cdot)$, $\Phi(\cdot)$ and $\Xi(\cdot)$:

$$\begin{aligned}
v(x) &= \chi \frac{x^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \\
u(x) &= \frac{x^{1-\sigma}-1}{1-\sigma} \\
\Psi(x) &= \kappa \nu x \left(\ln x - \ln \nu - 1\right) + \kappa \nu^2 \\
\Phi(x) &= a x^b - q \\
\Xi(x) &= \frac{\kappa_I}{2}\left(x-1\right)^2 \\
\Omega(x) &= N(e\mathbb{1}(\text{CBDC exists}) + fx).
\end{aligned}$$

Note that we allow the cost of CBDC function Ω to have both a fixed component that is turned on as soon as CBDC is implemented but does not vary with the amount of CBDC, and a component that varies with the amount of real CBDC as a function of GDP. We also multiply Ω by steady state labor for numerical convenience. The function Ψ is not exactly quadratic, but it has several useful properties described in Ulate (2021). Furthermore, its second order approximation around the steady state is:

$$\Psi(x) \approx^2 \frac{\kappa}{2}\left(x-\nu\right)^2,$$

which is the quadratic form that has been traditionally used in the literature.

We reiterate the equilibrium equations here according to their sector. Households (7 equations):

$$\begin{aligned}
\chi N_t^{\frac{1}{\eta}} &= C_t^{-\sigma}\frac{W_t}{P_t} \\
1 &= \beta(1+i_t)\mathbb{E}_t\left(\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{P_t}{P_{t+1}}\right) \\
\frac{1+i_t^{\mathcal{L}}}{1+i_t} &= ab\mathcal{L}_t^{b-1} \\
(1+i_t^{\mathcal{L}})^{\theta+1} &= \gamma_m + \gamma_d(1+i_t^d)^{\theta+1} + \gamma_{cbdc}(1+i_t^{cbdc})^{\theta+1} \\
m_t &= \gamma_m\left(\frac{1}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t \\
d_t &= \gamma_d\left(\frac{1+i_t^d}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t \\
cbdc_t &= \gamma_{cbdc}\left(\frac{1+i_t^{cbdc}}{1+i_t^{\mathcal{L}}}\right)^{\theta}\mathcal{L}_t.
\end{aligned}$$

Intermediate good firms (8 equations):

$$\begin{aligned}
Y_t^m &= A_t K_t^{\alpha} N_t^{1-\alpha} \\
\frac{W_t}{P_t} &= (1-\alpha)\frac{P_t^m}{P_t}\frac{Y_t^m}{N_t}
\end{aligned}$$

$$
\begin{aligned}
z_t &= \mathbb{E}_t\left(\alpha\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{P^m_{t+1}}{P_{t+1}}\frac{Y^m_{t+1}}{K_{t+1}}\right)\\
z_t &= \left(\psi(z^P_t)^{1-\theta^k}+(1-\psi)(z^{NP}_t)^{1-\theta^k}\right)^{\frac{1}{1-\theta^k}}\\
z^P_t &= \frac{Q_t}{P_t}\frac{1+i^l_t}{1+i_t}-(1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right)\\
z^{NP}_t &= \frac{Q_t}{P_t}\frac{1+i_t+\varrho}{1+i_t}-(1-\delta)\mathbb{E}_t\left(\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\right)\\
K^P_{t+1} &= \psi\left(\frac{z^P_t}{z_t}\right)^{-\theta^k}K_{t+1}\\
K^{NP}_{t+1} &= (1-\psi)\left(\frac{z^{NP}_t}{z_t}\right)^{-\theta^k}K_{t+1}.
\end{aligned}
$$

Capital producers (2 equations):

$$
\begin{aligned}
K^{NP}_{t+1}+K^P_{t+1} &= (1-\delta)\left[K^{NP}_t+K^P_t\right]+I_t\left(1-\Xi\left(\frac{I_t}{I_{t-1}}\right)\right)\\
1 &= \frac{Q_t}{P_t}\left[1-\Xi\left(\frac{I_t}{I_{t-1}}\right)-\Xi'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right]\\
&+ \mathbb{E}_t\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{Q_{t+1}}{P_{t+1}}\Xi'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2.
\end{aligned}
$$

Banks (10 equations):

$$
\begin{aligned}
\omega^d_{\mathcal{L},t} &= \gamma_d\left(\frac{1+i^d_t}{1+i^{\mathcal{L}}_t}\right)^{\theta+1}\\
\epsilon^d_t &= \frac{n-1}{n}\varphi^d+\frac{1}{n}\left[(1-\omega^d_{\mathcal{L},t})\theta+\frac{\omega^d_{\mathcal{L},t}}{b-1}\right]\\
1+i^d_t &= \frac{\epsilon^d_t}{\epsilon^d_t+1}(1+i_t-\mu^d)\\
\omega^{K_{NP}}_{K,t} &= \psi\left(\frac{z^P_t}{z_t}\right)^{1-\theta^k}\\
\epsilon^l_t &= \left\{\frac{n-1}{n}\varphi^l+\frac{1}{n}\left[(1-\omega^{K_{NP}}_{K,t})\theta^k+\frac{\omega^{K_{NP}}_{K,t}}{1-\alpha}\right]\right\}\frac{Q_t}{P_t}\frac{1+i^l_t}{1+i_t}\frac{1}{z^P_t}\\
1+i^l_t &= \frac{\epsilon^l_t}{\epsilon^l_t-1}\left[1+i_t+\mu^l+\kappa\nu\left(\ln\left(\frac{L_t}{F_t}\right)-\ln(\nu)\right)\right]\\
\frac{L_t}{P_t} &= \frac{Q_t}{P_t}K^P_{t+1}\\
\frac{X_t}{P_t}\frac{P_t}{P_{t-1}} &= i_{t-1}\frac{F_{t-1}}{P_{t-1}}+(i^l_{t-1}-\mu^l-i_{t-1})\frac{L_{t-1}}{P_{t-1}}+(i_{t-1}-\mu^d-i^d_{t-1})\frac{D_{t-1}}{P_{t-1}}\\
&- \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)\frac{F_{t-1}}{P_{t-1}}-\frac{F_{t-1}}{P_{t-1}}(1-\varsigma)\pi_t\\
\frac{F_t}{P_t} &= \frac{F_{t-1}}{P_{t-1}}(1-\varsigma)+\omega\frac{X_t}{P_t}
\end{aligned}
$$

$$\frac{H_t}{P_t} = \frac{F_t}{P_t} + \frac{D_t}{P_t} - \frac{L_t}{P_t}.$$

Retail firms (6 equations):

$$\begin{aligned}
1 &= (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{1-\phi} + \gamma\left(\frac{P_{t-1}}{P_t}\right)^{1-\phi} \\
\phi\Gamma_t^1 &= (\phi-1)\Gamma_t^2 \\
\Gamma_t^1 &= C_t^{-\sigma}\frac{P_t^m}{P_t}Y_t + \gamma\beta\mathbb{E}_t\left(\frac{P_t}{P_{t+1}}\right)^{-\phi}\Gamma_{t+1}^1 \\
\Gamma_t^2 &= C_t^{-\sigma}\frac{P_t^*}{P_t}Y_t + \gamma\beta\mathbb{E}_t\frac{P_t^*/P_t}{P_{t+1}^*/P_{t+1}}\left(\frac{P_t}{P_{t+1}}\right)^{1-\phi}\Gamma_{t+1}^2 \\
Y_t^m &= Y_t v_t^p \\
v_t^p &= \gamma\left(\frac{P_{t-1}}{P_t}\right)^{-\phi} v_{t-1}^p + (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{-\phi}.
\end{aligned}$$

Others (5 equations):

$$\begin{aligned}
Y_t &= C_t + I_t + G_t + \Gamma_t \\
\Gamma_t &= \mu^l\frac{L_{t-1}}{P_t} + \mu^d\frac{D_{t-1}}{P_t} + \varsigma\frac{F_{t-1}}{P_t} + \Psi\left(\frac{L_{t-1}}{F_{t-1}}\right)\frac{F_{t-1}}{P_t} + \varrho\frac{Q_{t-1}}{P_t}K_t^{NP} \\
&+ \Phi(\mathcal{L}_t) - \frac{M_t + D_t + CBDC_t}{P_t} + \Omega\left(\frac{CBDC_t}{P_tY_t}\right) \\
i_t &= (1-\rho_i)\left(\bar{\imath} + \psi_\pi(\pi_t - \overline{\pi})\right) + \rho_i i_{t-1} + \epsilon_t^i \\
A_t &= A_{t-1}^{\rho_a}\exp(\epsilon_t^a) \\
G_t &= gY_t.
\end{aligned}$$

Plus a value for the interest rate on CBDC (this would be -100% in the pre-CBDC scenario, but something like $i_t^{cbdc} = 0$ or $i_t^{cbdc} = i - 1$ in the post-CBDC scenario).

Appendix B.8 Welfare Change Measure

We define the (multiplicative) consumption equivalent variation required to keep the representative household indifferent between an initial scenario (for example the pre-CBDC deterministic steady state) and a new scenario (for example the post-CBDC deterministic steady state) to be the scalar ζ that satisfies the following equation:

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[u(C_t^{POST}) - v(N_t^{POST})\right] = \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[u(\zeta C_t^{PRE}) - v(N_t^{PRE})\right].$$

For example, if the scalar, ζ, that satisfies the previous equation comes out to be 1.0030, this indicates that the representative household needs to be given 0.3% of its initial-scenario consumption path to be indifferent between the initial and final scenarios. In the case where $u(\cdot) = \ln(\cdot)$, and when we are comparing two steady states, the previous equation becomes:

$$\ln(\overline{C}^{POST}) - v(\overline{N}^{POST}) = \ln(\zeta\overline{C}^{PRE}) - v(\overline{N}^{PRE})$$

$$\zeta = \exp\{[\ln(\overline{C}^{POST}) - v(\overline{N}^{POST})] - [\ln(\overline{C}^{PRE}) - v(\overline{N}^{PRE})]\}.$$

In our exposition, when comparing the pre-CBDC and the post-CBDC steady states, we refer to $(\zeta - 1) \cdot 100$ as the "welfare change from CBDC introduction".

Appendix C Additional Results and Robustness

Appendix C.1 Varying the Importance of CBDC

As discussed in Section 4.2, our calibration of the γ parameters governing the importance of the three liquid instruments is based on three moments. First, the three γ's sum to one: $\gamma_m + \gamma_d + \gamma_{cbdc} = 1$. Second, the deposits-to-liquidity ratio is equal to 0.8 in the pre-CBDC steady state. Third, from surveys of the potential use of CBDC we obtain that cash and CBDC would be used roughly to similar extents if CBDC paid an interest rate of zero, which implies $\gamma_{cbdc} = \gamma_m$. Out of these three moments, the least settled one is the latter one, as it is not based on "actual data" but is based instead on answers of households that might have limited information on how CBDC will operate. Therefore, in this subsection, we analyze how our results change if we modify the third targeted moment from $\gamma_{cbdc}/\gamma_m = 1$ to $\gamma_{cbdc}/\gamma_m = k$, where k is a parameter that can take values between 0.25 (so that CBDC is a fourth as important as cash) to 2 (so that CBDC is twice as important as cash). Notice that, when changing this moment, we have to recalibrate all of our internally-calibrated parameters to still match all of our targets.

Figure C.1 presents the results for our baseline economy with a long run steady-state policy rate of 2%. The k representing the target ratio γ_{cbdc}/γ_m is given on the x-axis, and several variables of interest are presented on the y-axis. The welfare change from introducing CBDC with its welfare-maximizing interest rate is given by the solid blue line, the deposits-to-GDP ratio is the orange dash-dotted line, the CBDC-to-GDP ratio is the dashed yellow line, and the welfare-maximizing CBDC rate is the purple line with circular markers. Certainly, the ratio of γ_{CBDC}/γ_m matters for the welfare gains of introducing CBDC, as is to be expected, but this dependence seems to be roughly linear, i.e., the gains for $k = 0.25$ are roughly a fourth of those for $k = 1$. For example, for $k = 1$, the welfare gains of introducing CBDC are 27 basis points, consistent with our baseline when choosing the welfare-maximizing CBDC rate. For $k = 0.25$ the welfare gains are 7.5 basis points, very close to one fourth of the gains in the baseline. For $k = 2$ the gains are 44 basis points, not exactly double those in the baseline, but not too far off. The optimal CBDC rate is more stable with k, 0.7% per year for $k = 0.25$, passing through 0.8% per year for $k = 1$ (replicating the baseline specification), and 0.9% for $k = 2$.

Appendix C.2 CBDC Introduction Effects Across Kappa and Theta

Figure C.2 plots the welfare change between the pre-CBDC scenario and the post-CBDC scenario for the baseline specification (where CBDC pays an interest rate of zero percent once it is introduced), for different levels of κ (the importance of bank equity for lending) and different levels of θ^k (the elasticity of substitution between pledgeable and non-pledgeable capital). As the importance of bank equity for lending increases, the welfare gain from introducing CBDC goes down. This makes sense because "disintermediating" banks, by lowering their profitability through the introduction of CBDC, decreases lending more when κ is high. Recall that our baseline value is $\kappa = 12$ basis points.

Figure C.1: This figure shows the percent welfare change from CBDC introduction, in the solid blue line, the deposits-to-GDP ratio, in the orange dash-dotted line, the CBDC-to-GDP ratio, in the yellow dashed line, and the welfare-maximizing CBDC rate, in annual percent, in the purple line with circular markers, for different levels of the ratio γ_{cbdc}/γ_m, which is given on the x-axis.

Across the different lines, we see that when θ^k is higher (the orange line), the welfare gains from introducing CBDC are higher (except for $\kappa = 0$). This is also to be expected, because when the substitutability between bank and nonbank intermediation is higher, firms can more easily switch between bank and nonbank borrowing when banks are disintermediated, and the detrimental aspects of CBDC introduction are muted (leading to higher overall welfare gains).

Figure C.3 displays the ratio of loan losses to deposits losses in the aggregate banking sector due to the introduction of CBDC. This ratio increases with κ, since a higher κ indicates that bank equity is more important for lending, and the CBDC-induced bank-profitability decline has more important implications for the loan rate and bank lending. Additionally, the higher the θ^k, the higher is the ratio of loan losses to deposit losses. This is due to the fact that, when θ^k is high, even a small increase in the loan rate leads firms to heavily substitute bank borrowing with nonbank borrowing.

Interestingly, even though the three lines in Figure C.3 for the ratio of loan losses to deposits losses are relatively far apart, indicating substantial differences in the intensity of the bank disintermediation effect across levels of θ^k, the three lines in Figure C.2 for the welfare change due to CBDC introduction are much closer together. Furthermore, the case of a high θ^k leads to higher welfare gains from CBDC introduction, despite the fact that it is associated with a higher bank-disintermediation effect. This is due to the counteracting effects of changing θ^k. On the one hand, a high θ^k means that any small deviation in the interest rate on bank loans from that on corporate bonds leads to a large fall in bank lending (a large bank disintermediation effect). On the other hand, a high θ^k also implies that firms are very adept at substituting bank and nonbank borrowing, so a given bank disintermediation effect has a smaller impact on aggregate

Figure C.2: This figure shows the welfare change (gain if positive, loss if negative) from CBDC introduction, in percent, for different levels of κ (the cost of deviating from the target loan-to-equity ratio) and three different levels of the elasticity of substitution between pledgeable and non-pledgeable capital.

capital, output, and welfare. This logic highlights the fact that the ratio of loan losses to deposit losses due to the introduction of CBDC is not the most important object to focus on when assessing the welfare implications of CBDC introduction.

Appendix C.3 Transition Between Steady States

Figures C.4 and C.5 depict the transition between the pre-CBDC and the post-CBDC steady state for several variables of interest. The transitions use our baseline calibration and a CBDC that pays an interest rate of zero percent. In orange, we have the initial steady state, in the dashed yellow line we have the final (post-CBDC) steady state, and in blue we have the transition between the two.

We can see that labor falls between the initial and final steady state, and it actually falls by more than that in the transition. Similarly, consumption increases between the initial and final steady state, and increases even more in the transition. This is possible because aggregate capital actually contracts in the new steady state (so the transition has disinvestment). Final output is lower in the new steady state, both due to the lower labor and lower capital. Nevertheless, consumption can end up higher because government spending, investment, and waste all fall in the new steady state, and allow consumption to be higher despite the lower final output.

Deposits and the share of deposits in liquidity all fall in the new steady state due to the introduction of CBDC, but not by much. The fraction of deposits in liquidity ($\omega^d_{\mathcal{L}}$) falls from 80% to 74%. The loan rate increases by around 12 basis points in the new steady state, as can be seen in the mid-right panel of Figure C.5, due to commercial banks having less equity, as can be seen in the bottom right panel of Figure C.5.

Figure C.3: This figure displays the ratio of loan losses to deposits losses in the aggregate banking sector due to CBDC introduction for different levels of κ (the cost of deviating from the target loan-to-equity ratio) and three different levels of the elasticity of substitution between pledgeable and non-pledgeable capital (θ^k).

Both the deposit rate and the rate on liquidity increase substantially in the new steady state as can be seen from the bottom row of Figure C.4.

Importantly, even though banks pay a higher deposit rate (due to the greater competition with CBDC) and they have less equity in the new steady state, they also charge a higher loan rate and pay less operating costs in the new steady state (due to having less equity, recall that their operating costs are given as a fraction of their equity). Overall, their return on equity is essentially unchanged between the initial and final steady state. This alleviates concerns that our model is missing an entry margin in response to changes in bank profitability that could potentially change the results. Overall, labor falls by 0.18%, and consumption increases by around 0.04%. Overall, welfare is approximately 22 basis points higher in the post-CBDC steady state than in the initial (pre-CBDC steady state).

Appendix C.4 Robustness to Recalibrating Additional Parameters

Sections 5.2 and 5.3 analyzed several CBDC-related outcomes for different levels of the policy rate. In those sections, only the discount factor, β, was changing to generate the different levels of the policy rate, and no other underlying parameters were changing along with it. In this section, along with the discount factor, we vary additional parameters to continue to match some targets which we matched in our baseline calibration. Namely, we recalibrate the values of the disutility of labor parameter χ, the q parameter in the liquidity-cost function Φ, the exogenous elasticity of substitution between different banks in loans φ^l, the managerial cost of operating the bank ς, and the fraction of bank profits that stay in the bank ω. We do so to continue to match the following targets in different steady states associated with different policy-rate levels:

Figure C.4: This figure depicts the transition (under perfect foresight) between the pre-CBDC steady state and the post-CBDC steady state for several variables of interest. CBDC pays an interest rate of 0% and we use the baseline calibration.

1) labor is equal to one third, 2) $\Phi(\cdot) = m + d + cbdc$, 3) the endogenous share of loans in firm borrowing is equal to the exogenous share, ψ, 4) Banks are at their loan-to-equity ratio target, $L/F = \nu$, and 5) bank return on equity is 2.25% quarterly.

The results that we obtained in Sections 5.2 and 5.3 are qualitatively robust to this recalibration. Quantitatively, the results do change but to a fairly small degree. As an illustration, we reproduce Figure 5.3, but now with the recalibration of the aforementioned parameters. Figure C.6 provides the results. The orange dash-dot line for the policy rate and the yellow dashed line for the rule-of-thumb CBDC rate are still the same as those in Figure 5.3. The blue line is now different, and increases a bit more steeply with the policy rate, but the differences are fairly small. Further results with this recalibration procedure are available upon request.

Figure C.5: This figure continues Figure C.4, depicting the transition (under perfect foresight) between the pre-CBDC steady state and the post-CBDC steady state for additional variables of interest. CBDC pays an interest rate of 0% and we use the baseline calibration.

Appendix C.5 IRFs to a Technology Shock

Figure C.7 presents the IRFs of the economy to a 25-basis-points positive productivity shock, ϵ_t^a, with a persistence of 0.95 (see equation 3.26 for the law of motion of the technology shock). While the response of the economy to a technology shock is obviously different than the one to a monetary policy shock depicted in Figure 5.6, our main conclusion that responses to the shock are very similar across different remuneration schedules for CBDC is preserved.

Figure C.6: This figure displays the policy rate, in orange (in both axes, so it is the 45-degree line), the welfare-maximizing level of the CBDC rate, in blue, and an approximate welfare-maximizing rule-of-thumb rate which is the maximum between 0 and the policy rate minus 1%, in yellow.

Figure C.7: This figure depicts the IRFs to a 25 basis points positive productivity shock, with a persistence of 0.95, for different CBDC remuneration schedules.

Appendix C.6 Euro-Area Calibration

Table C.1: Euro Area Calibration

Param.	Value	Description	Target or source
Panel A. Nonbank			
β	0.9950	Discount factor	2% policy rate
χ	8.7428	Disutility of labor*	One-third S.S. labor
η	1.0000	Frisch elasticity	Chetty et al. (2011)
σ	1.0000	Inverse of the I.E.S.	Balanced Growth
α	0.3333	Capital share	Standard
δ	0.0200	Depreciation rate	8% annual dep.
κ_I	2.0000	Investment adjustment cost	Sims and Wu (2021)
ϕ	6.0000	Elasticity of subs. b/t diff. goods	20% mark-up
γ	0.7500	Prob. of keeping prices fixed	One-year duration
ψ_π	1.5000	Inflation coefficient, Taylor rule	Standard
ρ_i	0.8000	Smoothing parameter, Taylor rule	Standard
g	0.2000	Steady state G/Y	Standard
Panel B. Deposit side			
n	1.0863	Number of banks*	Deposit rate target #1
θ	675.10	E.o.S. between liquid instruments*	Deposit rate target #2
φ^d	852.59	E.o.S. between banks in deposits*	Deposit rate target #3
μ^d	-0.18%	Cost of issuing deposits*	Deposit rate target #4
γ_m	0.3379	Importance of cash in liquidity*	$\gamma_m + \gamma_d + \gamma_{cbdc} = 1$
γ_d	0.3242	Importance of deposits in liquidity*	$D/\mathcal{L} = 0.8$ at $i = 2\%$
γ_{cbdc}	0.3379	Importance of CBDC in liquidity*	$\gamma_{cbdc} = \gamma_m$ (Bidder et al.)
a	0.9294	Parameter in liquidity function Φ*	$\mathcal{L}/Y = 2.4$ quarterly
b	1.0375	Parameter in liquidity function Φ*	Estimation
q	-0.0926	Parameter in liquidity function Φ*	S.S. relationship
Panel C. Loan side			
ψ	0.8200	Importance of pledgeable capital*	Own calculations
ϱ	0.70%	Extra cost of corporate-bond borrowing	Schwert (2020)
μ^l	0.35%	Cost of issuing loans	Schwert (2020)
φ^l	90.503	E.o.S. between banks in loans*	$i^l = i + \varrho \Rightarrow \theta^k = f(\varphi^l)$
θ^k	5.0000	Subs. between NP and P capital	Feasible region
Panel D. Joint bank side			
ω	0.5886	Fraction staying in bank*	$L/F = \nu$ in S.S.
ς	0.0283	Bank managerial cost*	2.25% S.S. ROE
ν	9.0000	Loan-to-equity ratio target	Ulate (2021)
κ	0.0012	Cost of deviating from target ratio	Ulate (2021)

Notes: This table contains the parameter values used in our Euro-Area calibration, together with their description and their source or target. An asterisk in the "description" column indicates that the parameter has changed between the baseline calibration and the Euro-Area calibration. All interest rates are annualized.